

An Elementary Proof of Bott Periodicity

off Periodicity

$\tilde{K}(\varepsilon^2 x)$

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In class $\tilde{K}(X) \cong \tilde{K}(\Sigma^2 X)$

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Now: $K(X) \otimes K(S^2) \cong K(X \times S^2)$

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bundles

mentary Proof of Bott Periodicity

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Understand Vector bundles over

$X \times S^2$ in terms of bundles over $X$

# An Elementary Proof of Bott Periodicity

In class: $\tilde{K}(X) \cong \tilde{K}(\Sigma^2 X)$

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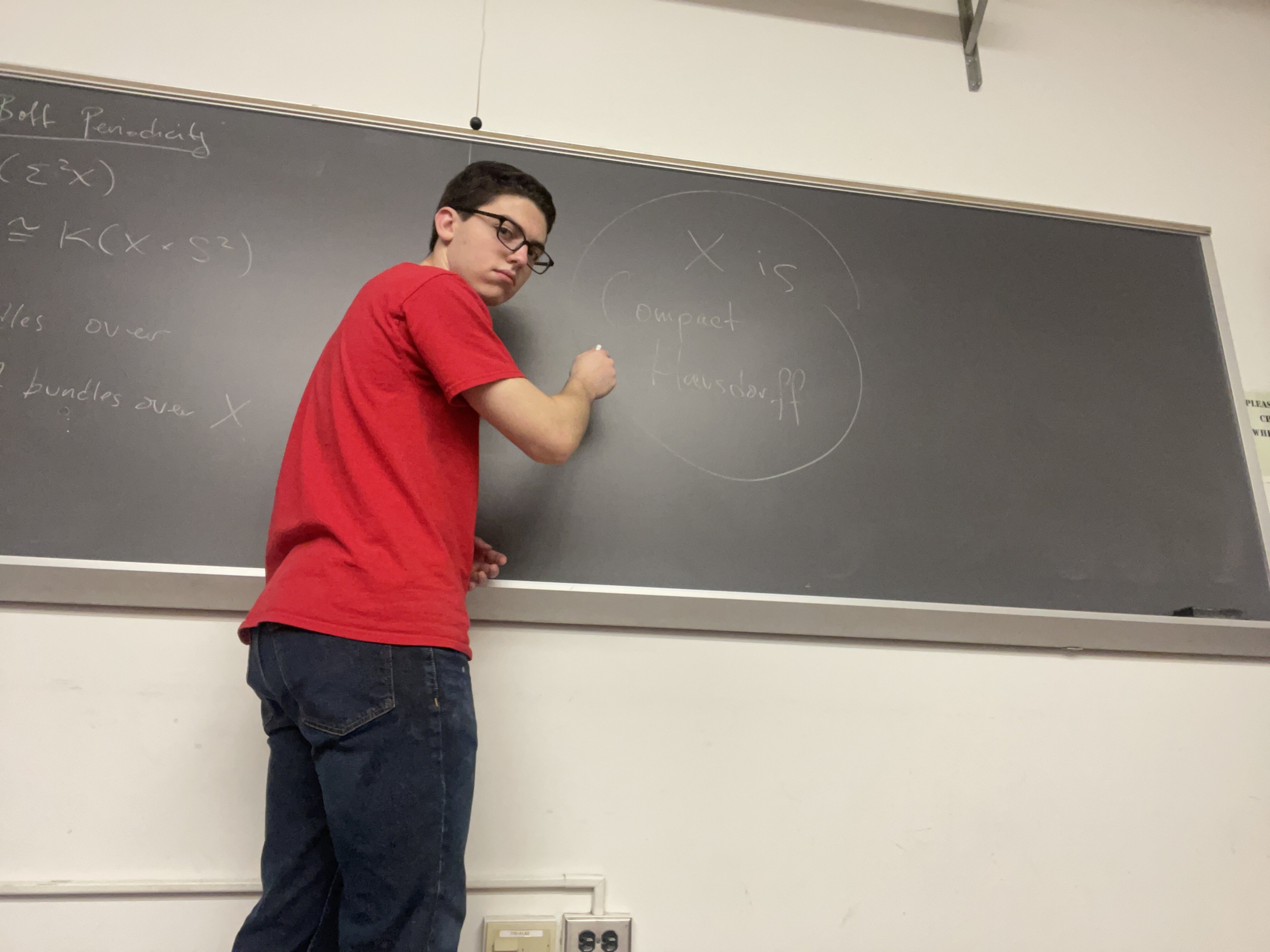
Understand Vector bundles over $X \times S^2$ in terms of bundles over $X$


Bott Periodicity
$(\Sigma^2 X)$
$\cong K(X \times S^2)$
dles over
bundles over X
X is
Compact
Hausdorff



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What is the relationship btwn

$X$ & $X \times S^2$ ?

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$E \xrightarrow{p} X$ is a Complex V.B.

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$\times S^2$)

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Def $P(E) \longrightarrow X$ is the Projective Bundle

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remove the 0-section & mod out by non-zero Scalars.

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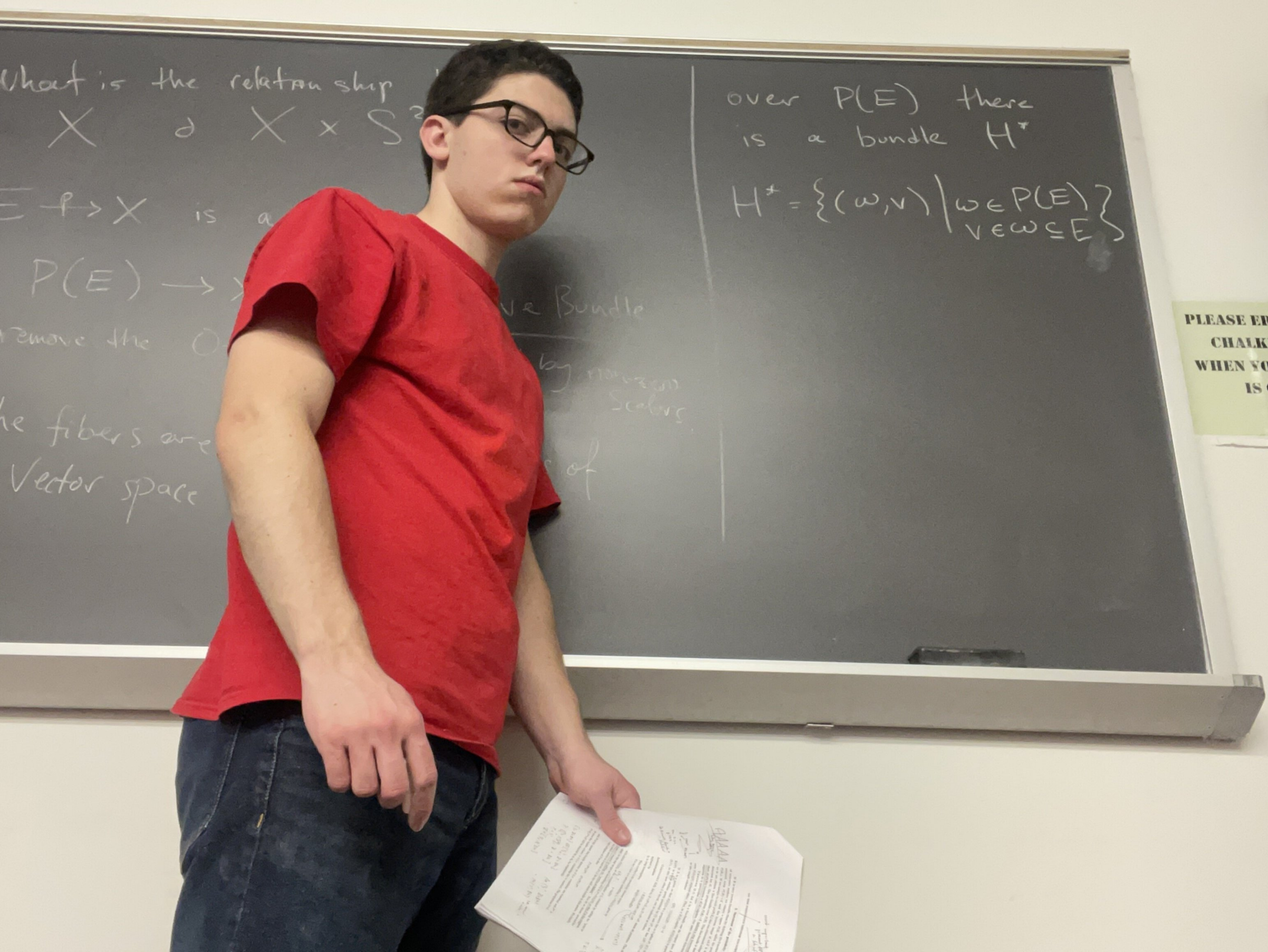
Def $P(E) \longrightarrow X$ is the Projective Bundle

remove the 0-section of & mod out by non-zero Scalars.

the fibers are the projective sections of Vector space fibers.


What is the relationship
X ∂ X × S²
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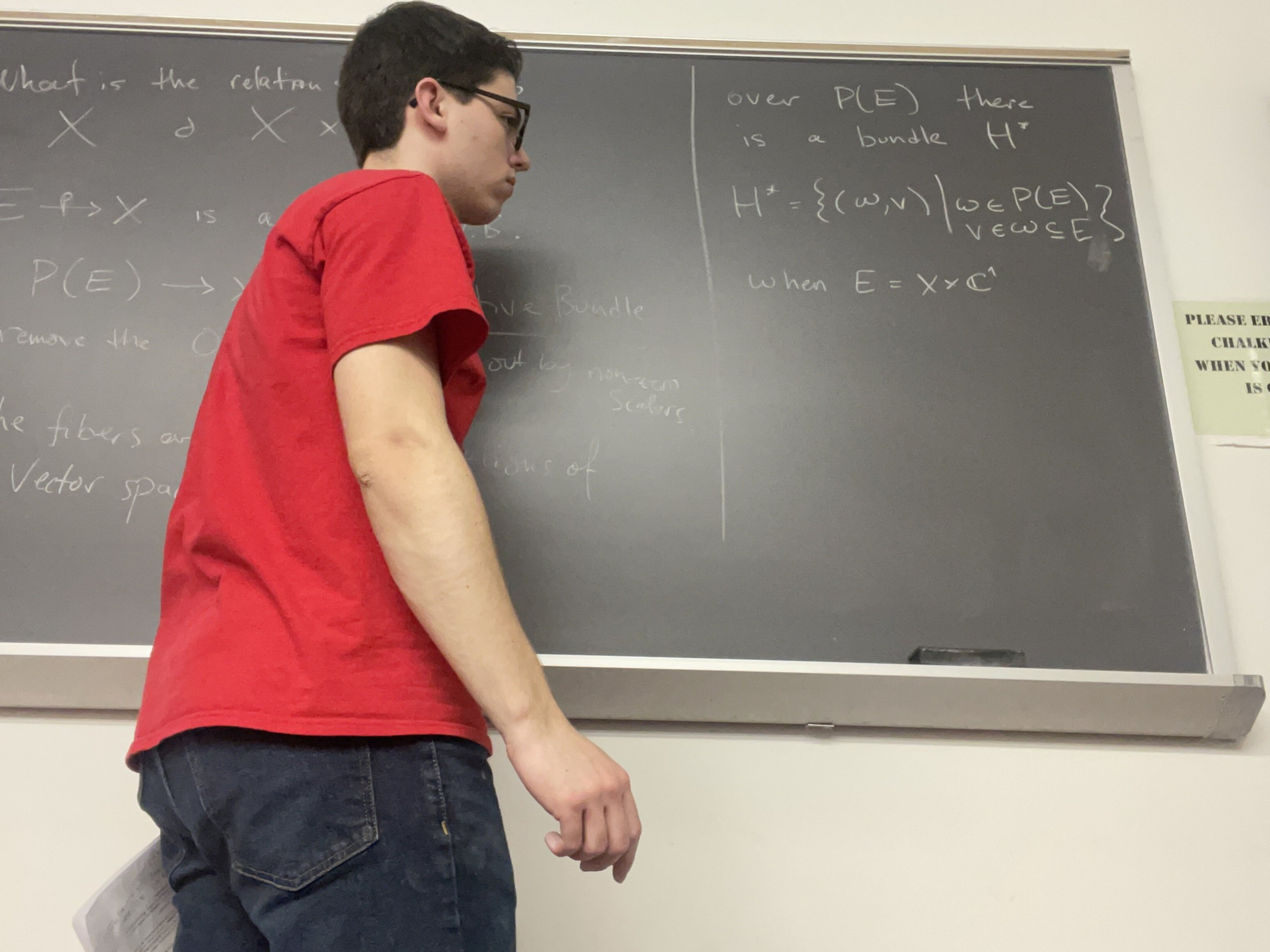
over P(E) there
is a bundle H*
H* = {(ω,v) | ω ∈ P(E), v ∈ ω ⊆ E}
PLEASE ER
CHALK
WHEN YO
IS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PLEASE ER
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What is the relationship b

$X \quad \partial \quad X \times S^2$

$E \xrightarrow{p} X$ is a C

$P(E) \longrightarrow X$

Remove the 0-sec

The fibers are th

Vector space f

e Bundle

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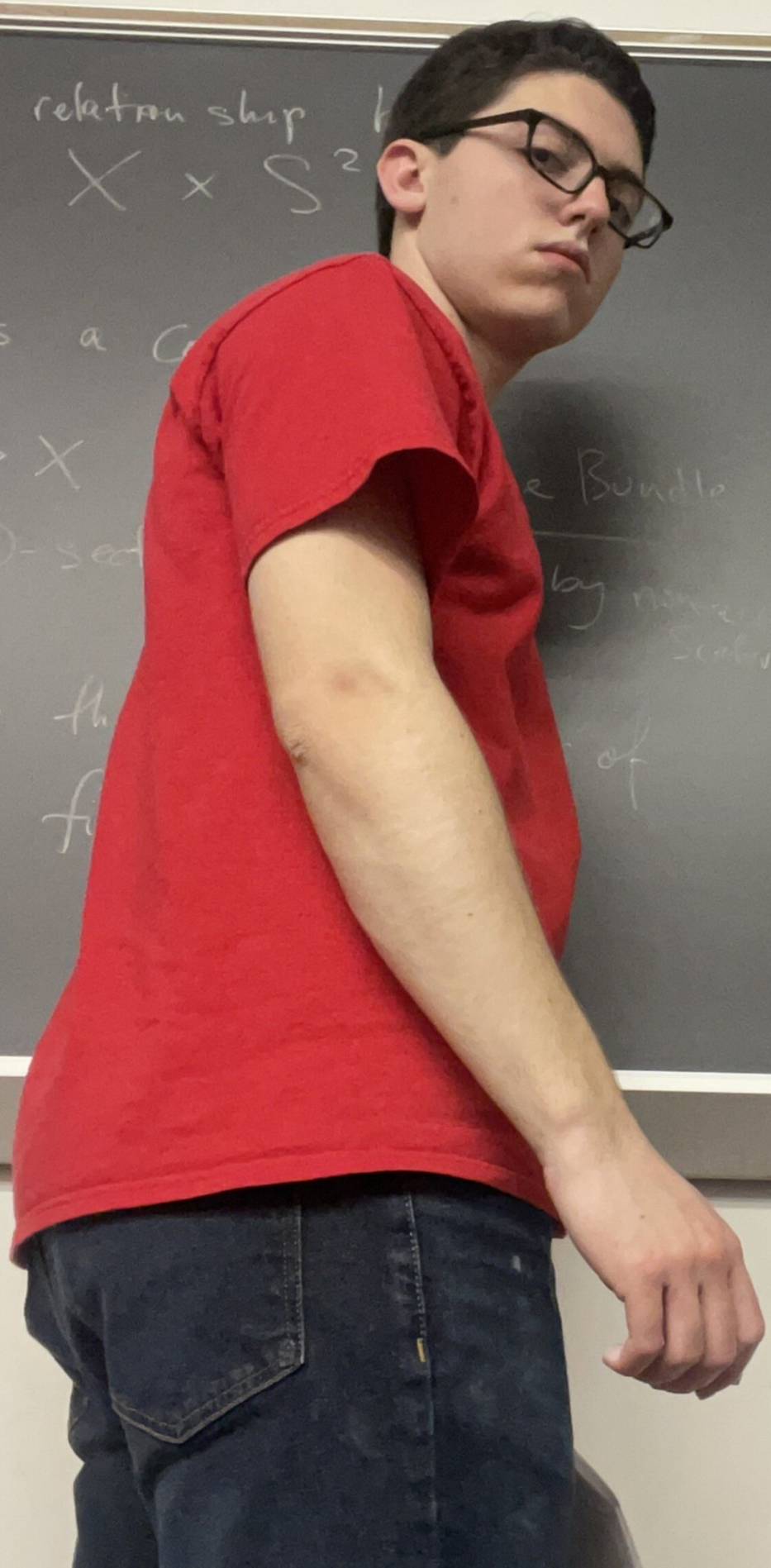
of

over $P(E)$ there is a bundle $H^*$

$H^* = \{(\omega, v) \mid \omega \in P(E),\ v \in \omega \subseteq E\}$

when $E = X \times \mathbb{C}^1 = \varepsilon^1$



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$$H^* = \{(\omega, v) \mid \omega \in P(E),\ v \in \omega \subseteq E\}$$

"$\gamma_{1,n}$"

PLEASE ER
CHALK
WHEN YO
IS

An Elementary Proof of Bott Periodicity

In class: $\tilde{K}(X) \cong \tilde{K}(\Sigma^2 X)$

Now: $K(X) \overset{\text{def}}{=} K(S^2) \cong K(X \times S^2)$

Thom Iso for K-Theory

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Thom Iso for K-Theory: $L$ a line bundle over $X$, $H$ over $P(L \oplus \varepsilon^1)$

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Iso: L a line bundle over X, H over $P(L \oplus \varepsilon^1)$

$K(P(L \oplus \varepsilon^1))$ is a $K(X)$-algebra

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Thom Iso for K-Theory: $L$ a line bundle over $X$, $H$ over $P(L \oplus \varepsilon^1)$

$K(P(L \oplus \varepsilon^1))$ is a $K(X)$-algebra generated by $[H]$ subject to

$$([H] - [\varepsilon^1])([L][H] - [1]) = 0$$

Bott Periodicity

$\tilde{\Sigma}(\Sigma^2 X)$

$\cong K(X$

H over P(

-algebra

)([L][H] -

What is the relationship btwn

$X$ & $X \times S^2$ ?

when $L = \varepsilon^1$

Bott Periodicity

$\tilde{K}(\Sigma^2 X)$

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What is the relationship btwn

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$\cong P(\varepsilon^2)$

$\cong X \times \mathbb{C}P^1$

$\cong X \times S^2$

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$\cong X \times \mathbb{CP}^1$

$\cong X \times S^2$ !

Bott Periodicity

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$\cong K(X \times$

H over $P(L$

-algebra

$)([L][H]$

What is the relationship btwn

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$\cong P(\varepsilon^2)$

$\cong X \times \mathbb{C}P^1$

$\cong X \times S^2$ !

$K(X) \otimes_{\partial} K(S^2) \xrightarrow{\sim} K(X \times S^2)$

as Rings

Bundles over $P = P(L \oplus \varepsilon^1)$

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note that $P$ is a compactification of $L$ by adding an "$\infty$-section"

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$P = P^\circ \cup P^\infty$

$S =$


Bundles over
note that P is a
of L by
S ⊆ L is
P = P
S = P

Bundles over $P = P(L \oplus \varepsilon^1)$

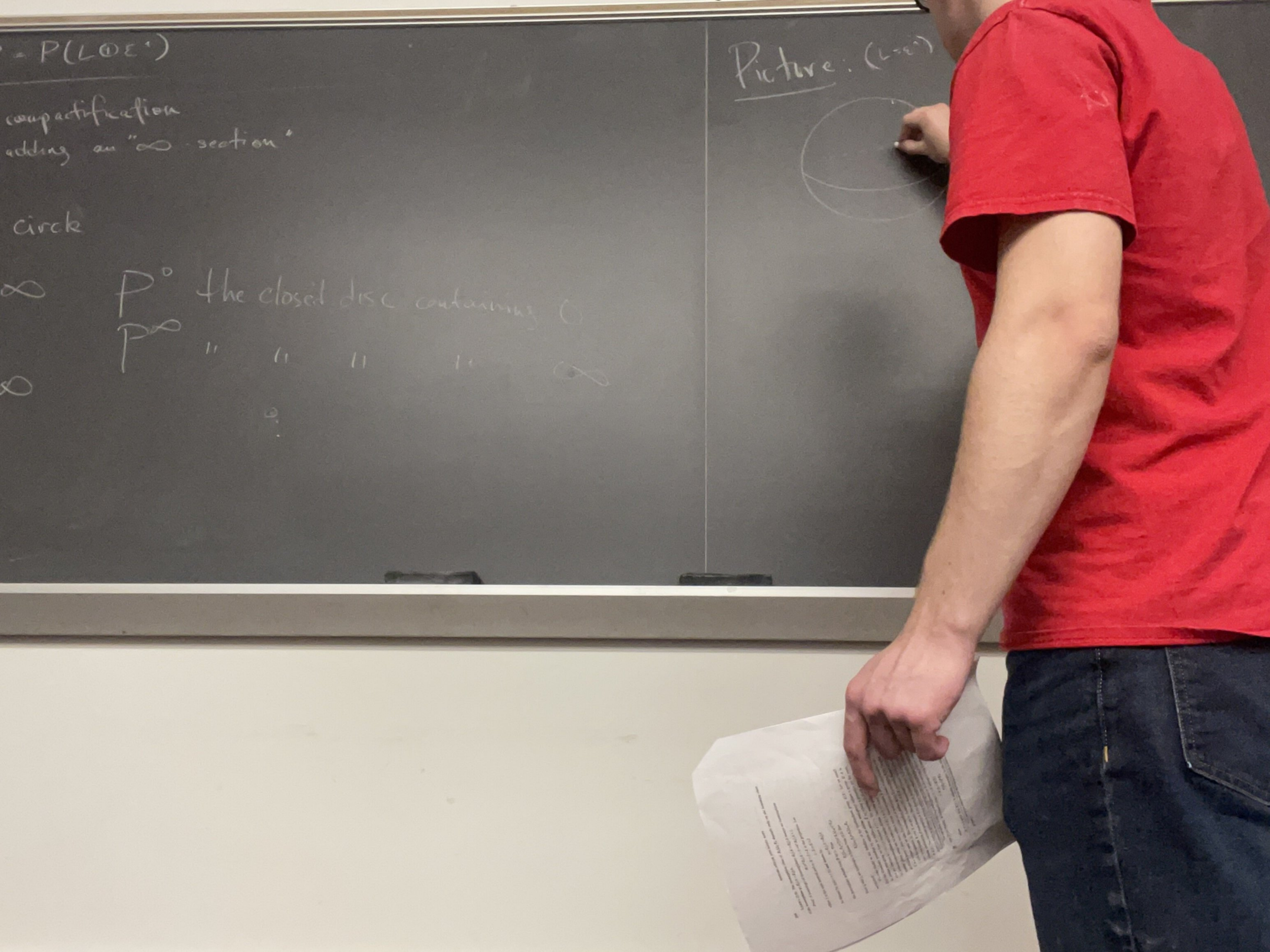
note that $P$ is a compactification of $L$ by adding an "$\infty$-section"

$S \subseteq L$ is the unit circle

$P = P^\circ \cup P^\infty$ $P^\circ$ the closed disc containing $0$

$P^\infty$ " " " " " $\infty$

$S = P^\circ \cap P^\infty$


= P(L⊕ε¹)
compactification
adding an "∞ section"
circle
∞
P⁰ the closed disc containing 0
P∞ " " " " ∞
∞
Picture: (L=ε¹)

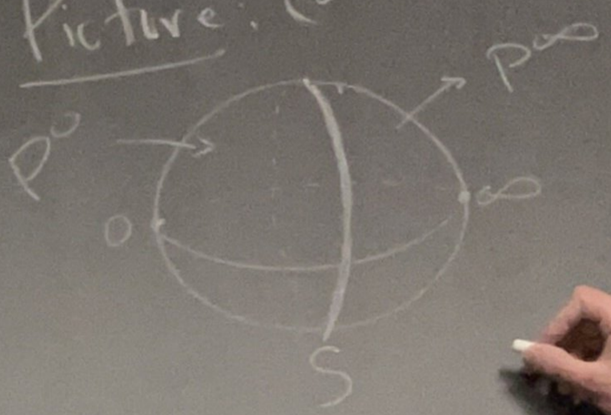
$= P(L \oplus \varepsilon^1)$

compactification
adding an "$\infty$ section"

circle

$\infty$ $P^0$ the closed disc containing 0
$P^\infty$ " " " " $\infty$
0

Picture: $(L-S^2)$



$= P(L \oplus \varepsilon^1)$

compactification
adding an "$\infty$-section"

circle

$\infty$ $P^0$ the cl
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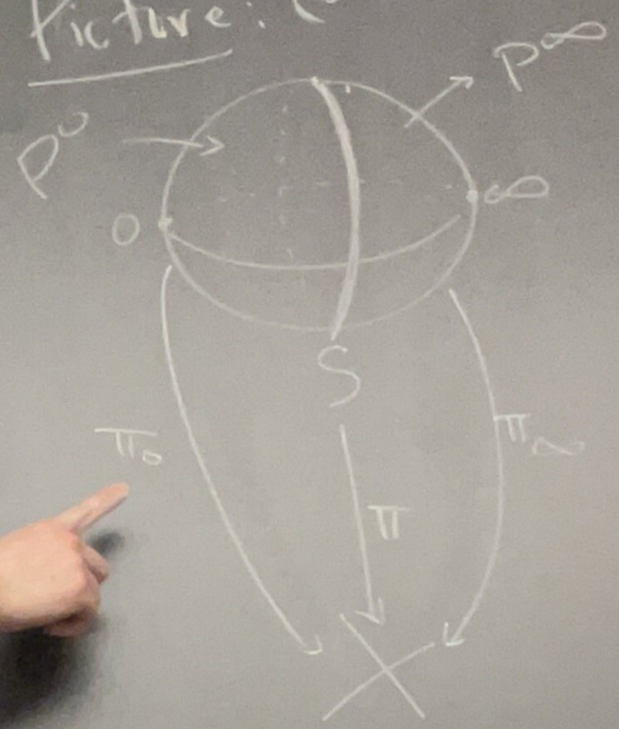
Picture: $(L = \varepsilon^1)$



$= P(L \oplus \varepsilon^1)$

compactification
adding an "$\infty$ section"

circle

$\infty$ $P^0$ the clos
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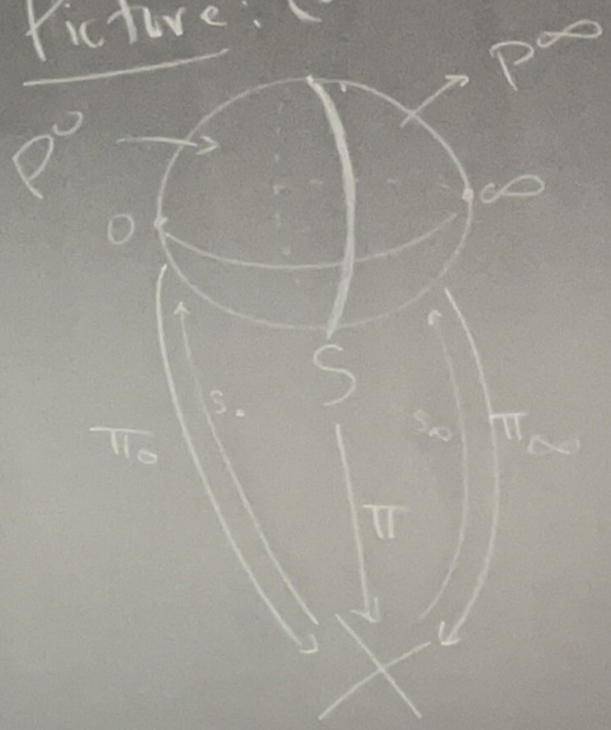
$\infty$

Picture: $(L-\varepsilon^1)$



Picture:

containing $\emptyset$

" $\infty$

Picture:

$E^0$ $E^\infty$

containing $\emptyset$

" $\infty$

Picture

$E^0$ $E^\infty$

containing $\emptyset$

" $\infty$

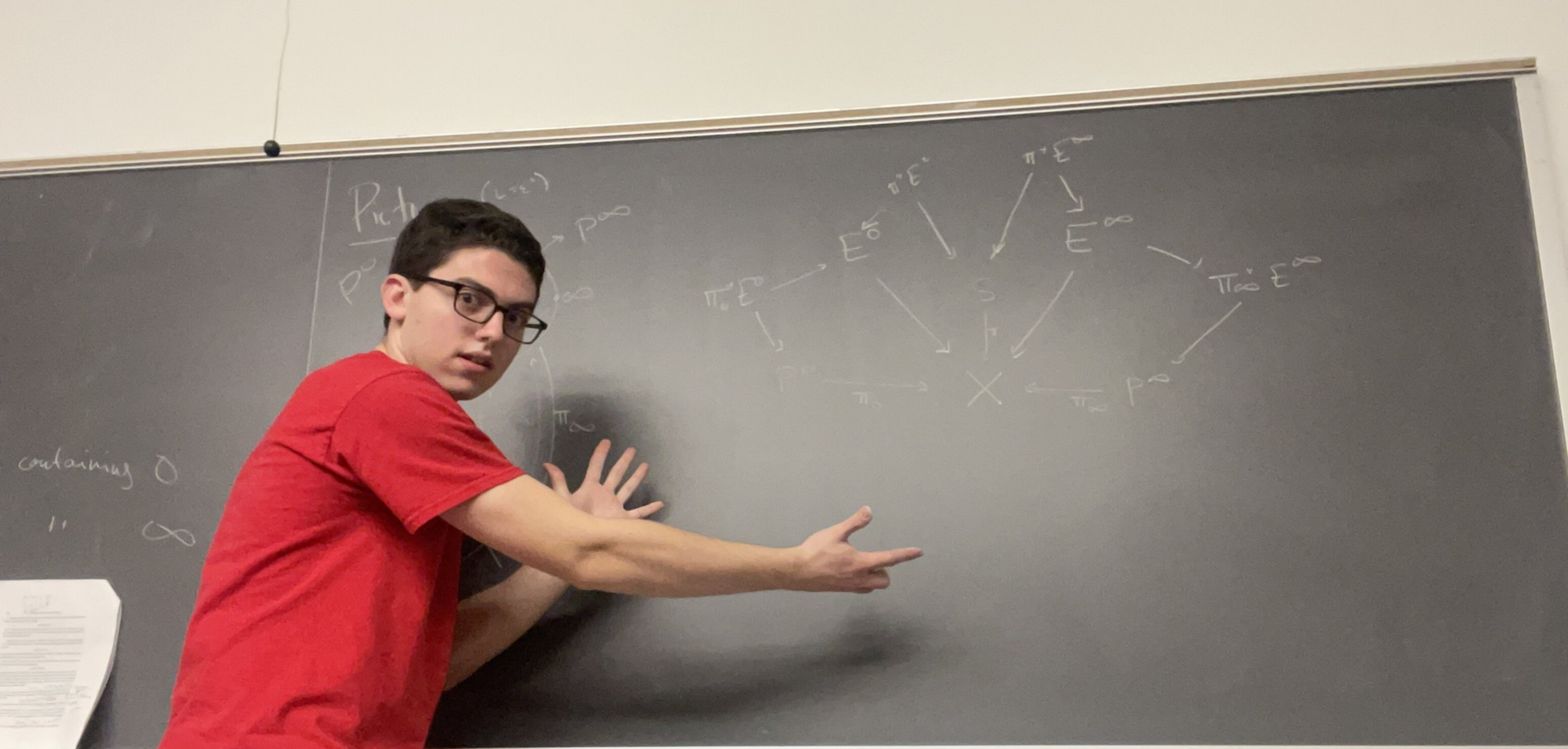
containing $\emptyset$

" $\infty$

$E^{\circ}$

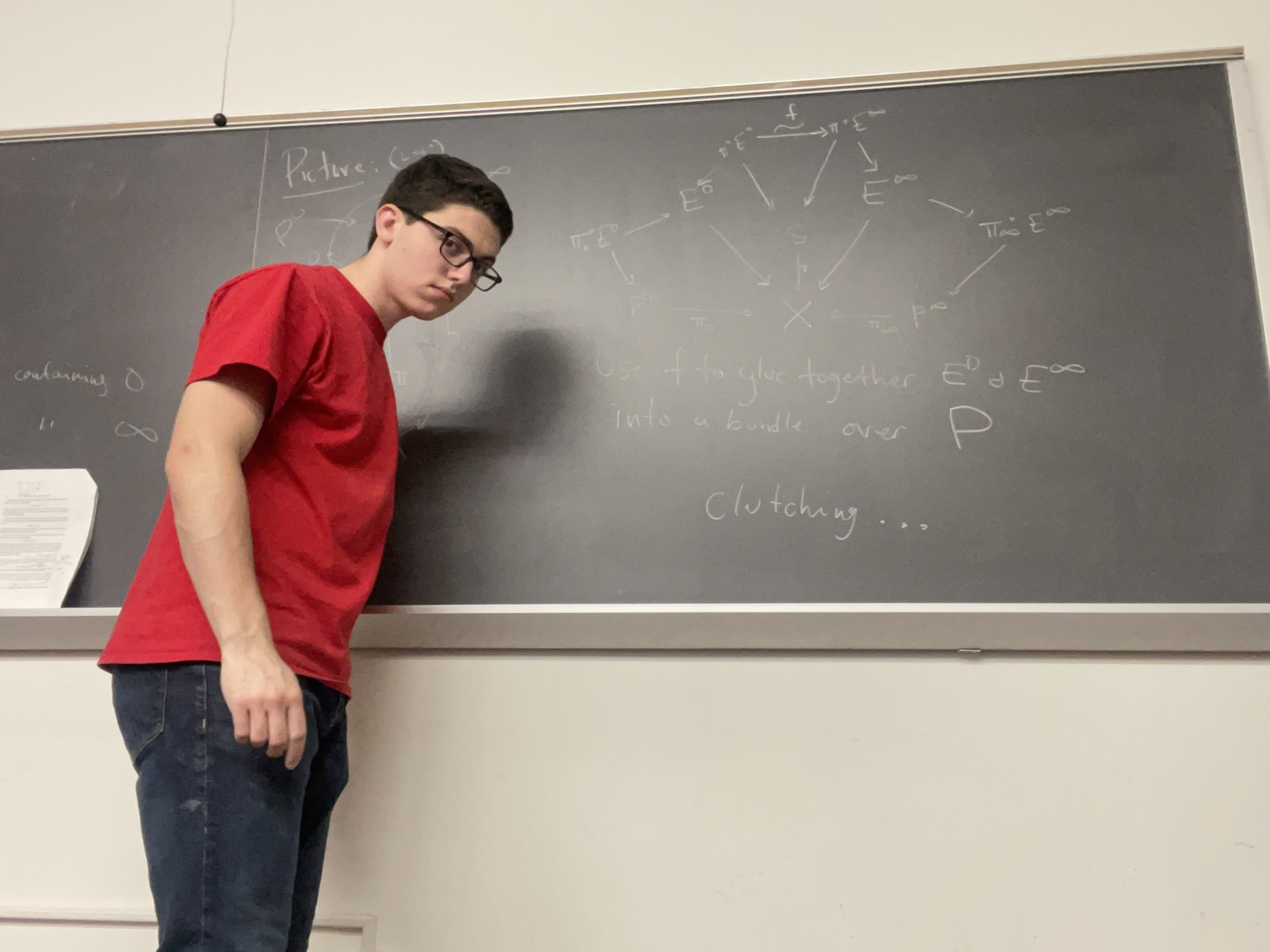
$E^{\infty}$

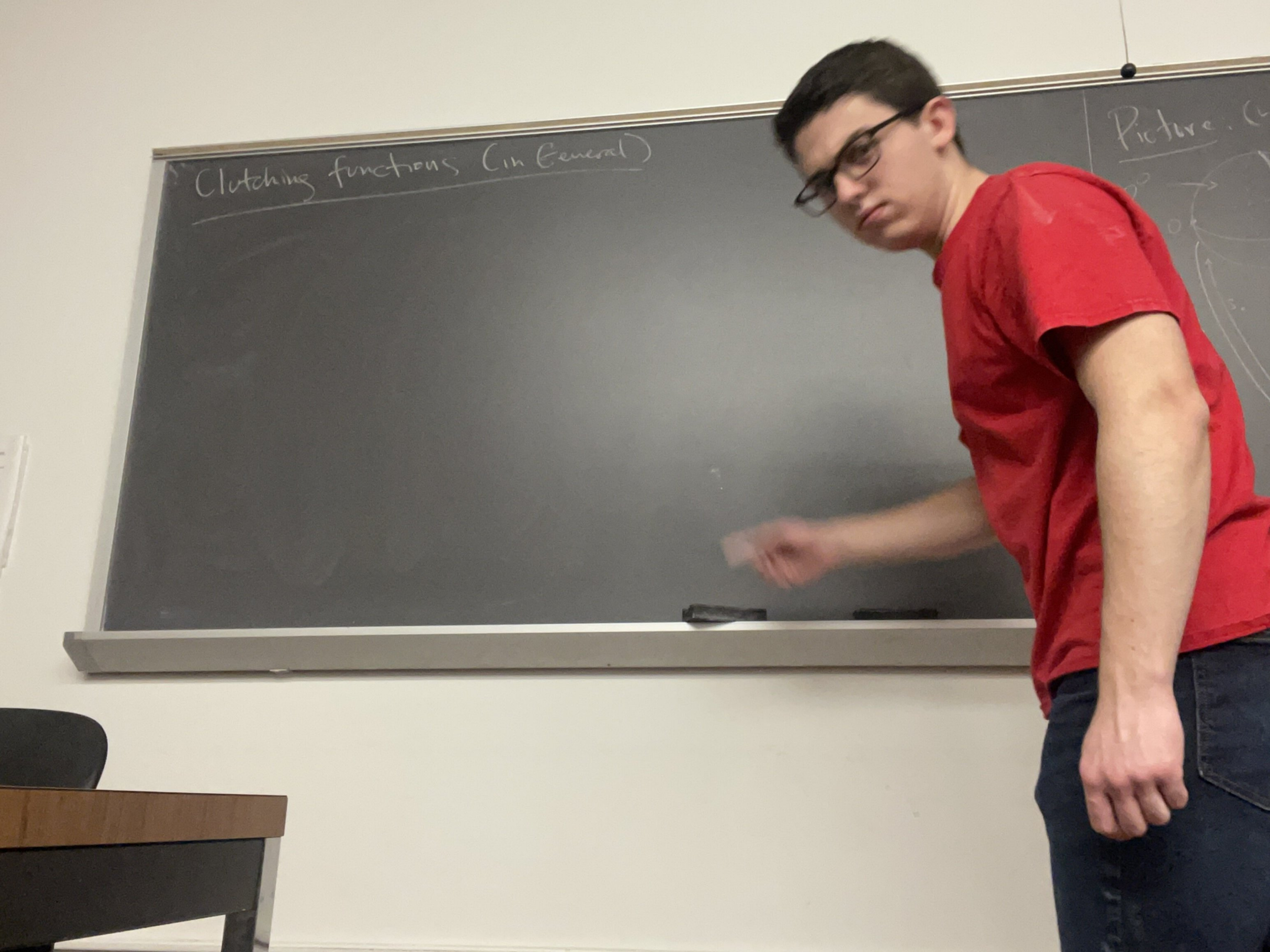

Picture:
containing 0
" ∞
E°
E∞
X


containing


containing ∅


containing 0
" ∞
Use f to glue together E^0 & E^∞
into a bundle over P


Picture:
containing 0
" ∞
Use f to glue together E^0 & E^∞
into a bundle over P
Clutching ...


Clutching functions (in General)
Picture:

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Picture:

## Clutching functions (in General)

$X = X_1 \cup X_2$

$A = X_1 \cap X_2$

Picture:

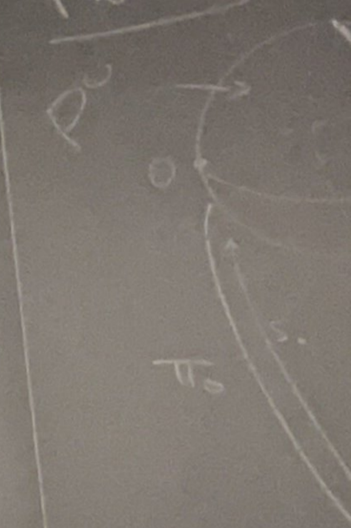
## Clutching functions (in General)

$$X = X_1 \cup X_2$$

$$A = X_1 \cap X_2$$

$$\begin{array}{cc} E_1 & E_2 \\ \downarrow & \downarrow \\ X_1 & X_2 \end{array}$$

Picture: (



## Clutching functions (in General)

$X = X_1 \cup X_2$

$= X_1 \cap X_2$

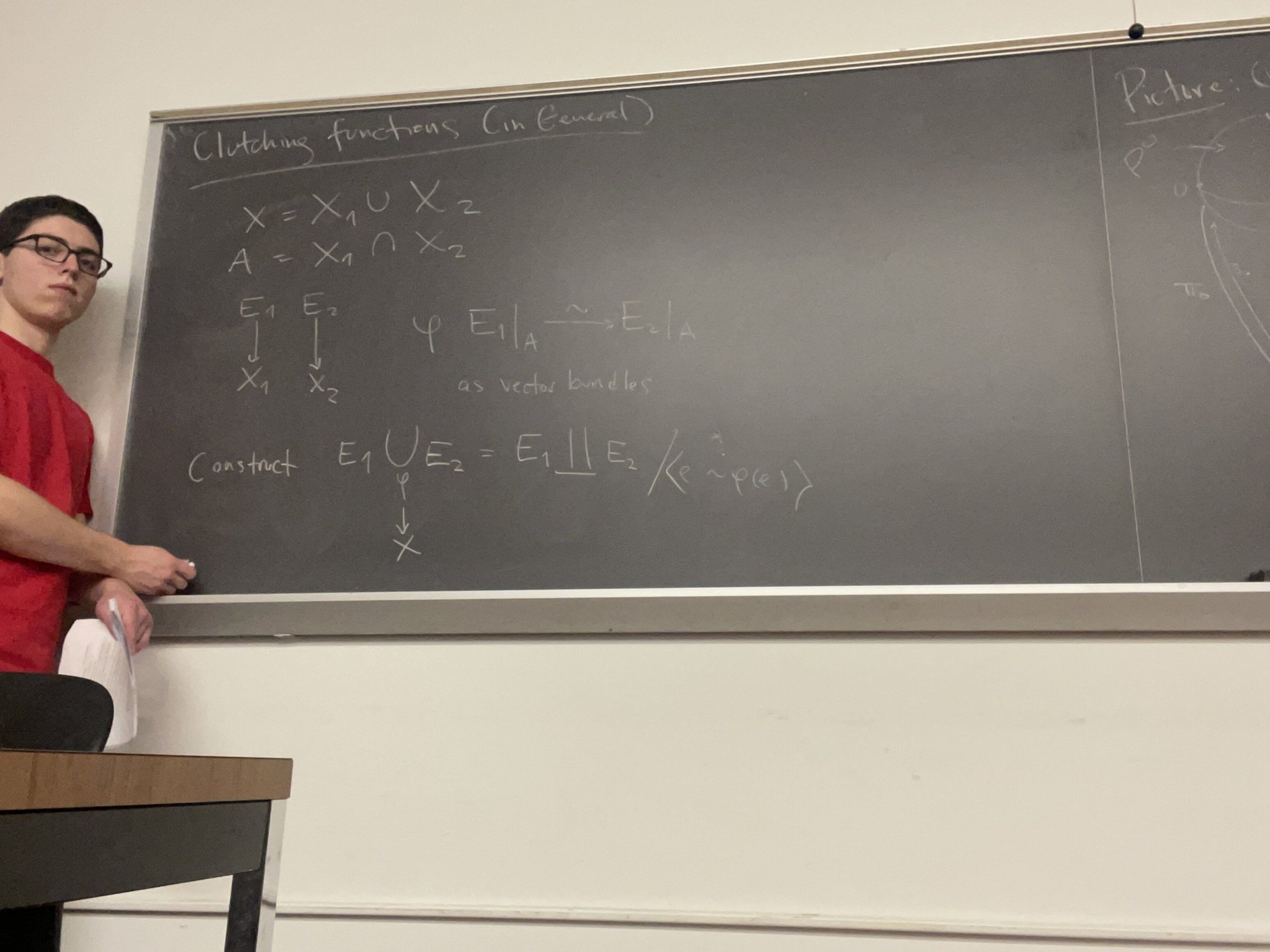
$E_1 \quad E_2$

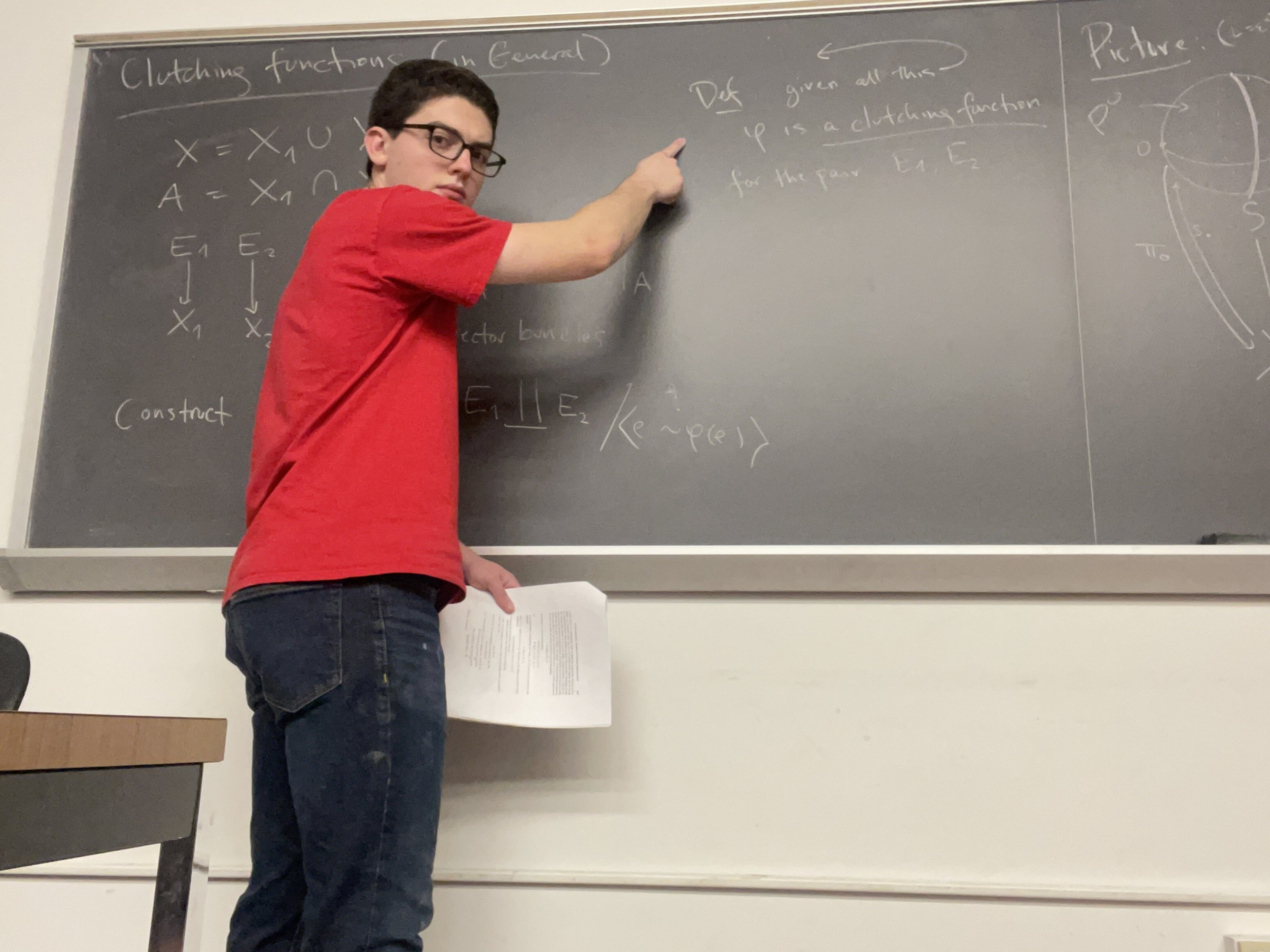
$\varphi \; E_1|_A \xrightarrow{\sim} E_2|_A$

$X_1 \quad X_2$

as vector bundles

Picture:


Clutching functions (in General)
$X = X_1 \cup X_2$
$A = X_1 \cap X_2$
$E_1 \to X_1$, $E_2 \to X_2$
$\varphi \; E_1|_A \xrightarrow{\sim} E_2|_A$
as vector bundles
Construct $E_1 \cup_\varphi E_2 = E_1 \amalg E_2 / \langle e \overset{?}{\sim} \varphi(e) \rangle$
$\varphi \to X$
Picture:
$\pi_0$


Clutching functions (in General)
X = X1 ∪
A = X1 ∩
E1 E2
X1 X2
A
ector bundles
Construct
E1 ⊔ E2 / ⟨e ~ φ(e)⟩
Def given all this
φ is a clutching function
for the pair E1, E2
Picture:
S
π0

# Clutching functions (in General)

Some important properties:

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1) $E \to X$, $E|_{X_1} \bigcup_{\text{clutching}} E|_{X_2} \cong E$

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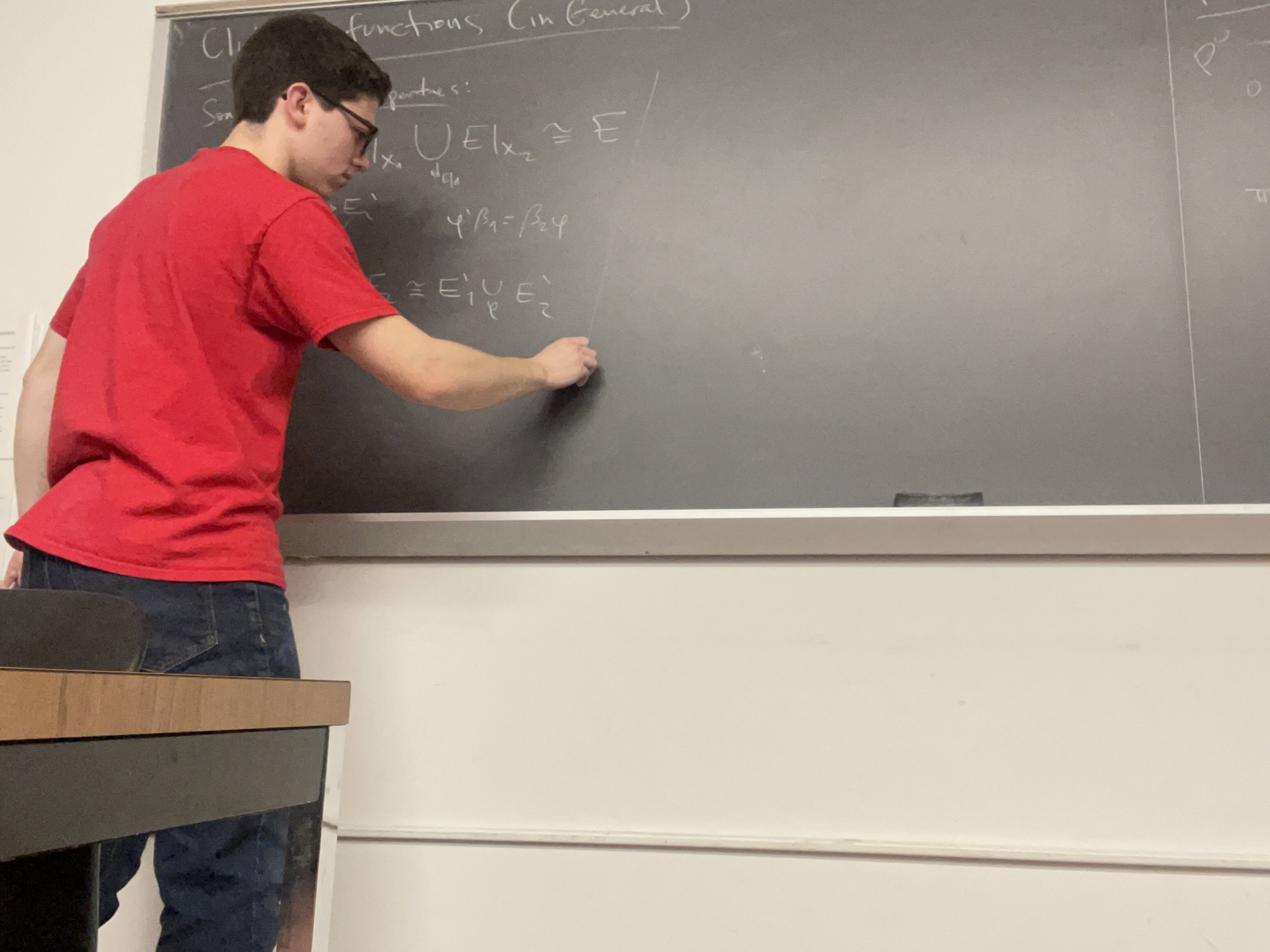
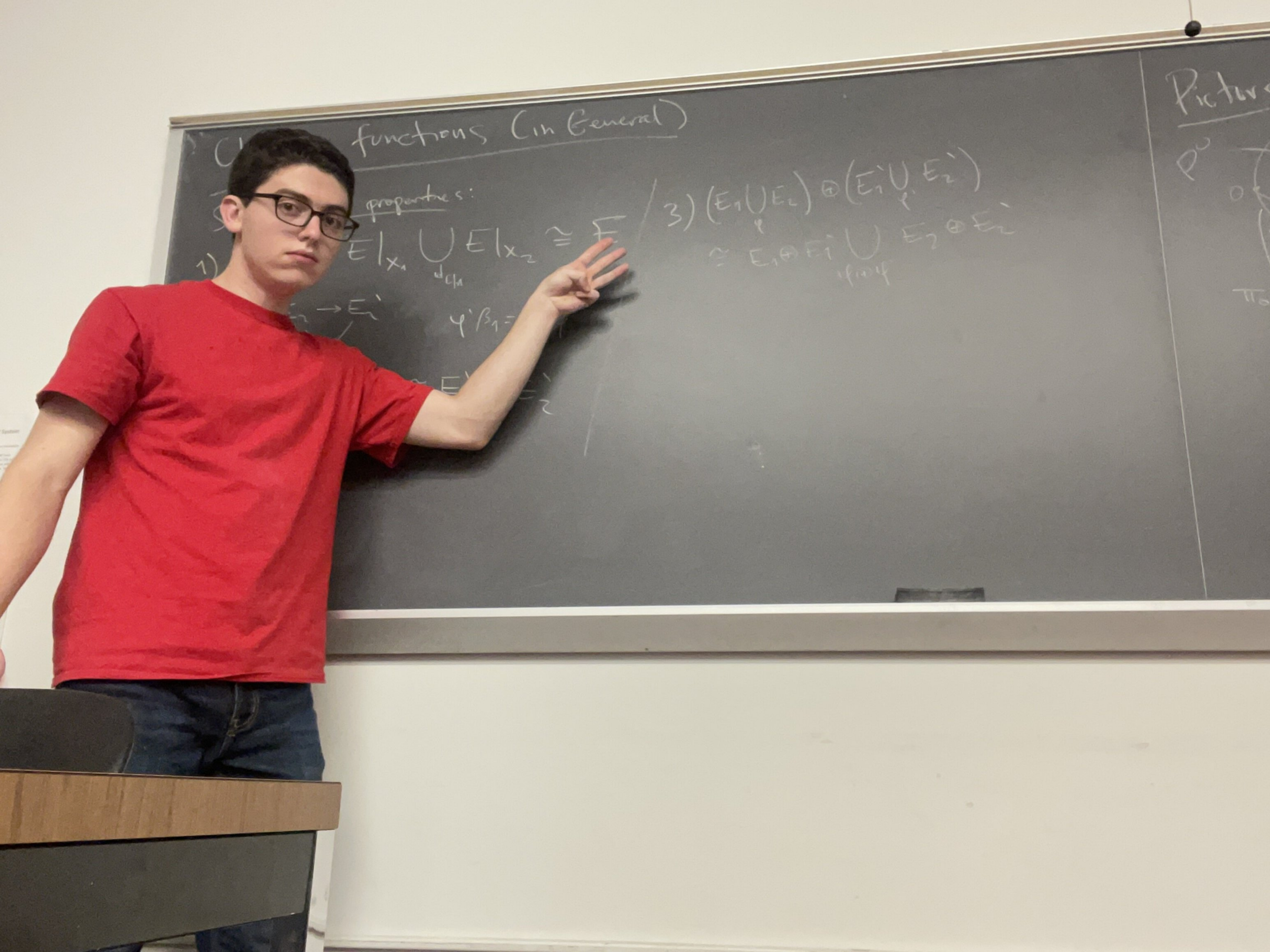
1) $E \to X$, $E|_{X_1} \bigcup_{\text{id}_{C_{12}}} E|_{X_2} \cong E$

2) $\beta_2$ $E_2 \to E_2'$ $\varphi' \beta_1 = \beta_2 \varphi$



then $E_1 \bigcup_{\varphi} E_2 \cong E_1' \bigcup_{\rho} E_2'$


functions (in General)
$\varphi'\beta_1 = \beta_2\varphi$
$\cong E_1' \cup_\varphi E_2'$


functions (in General)
properties:
1)
3)
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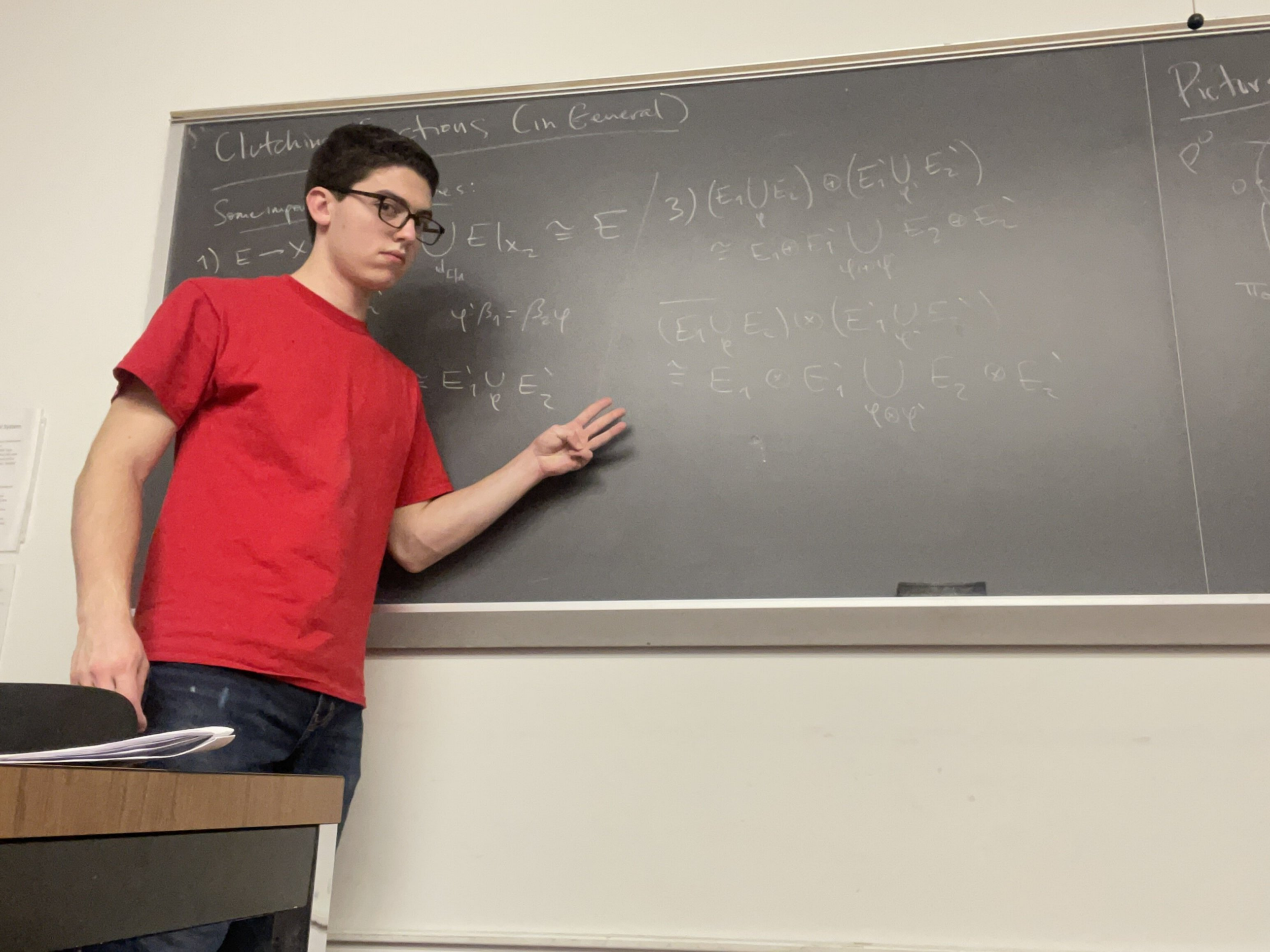
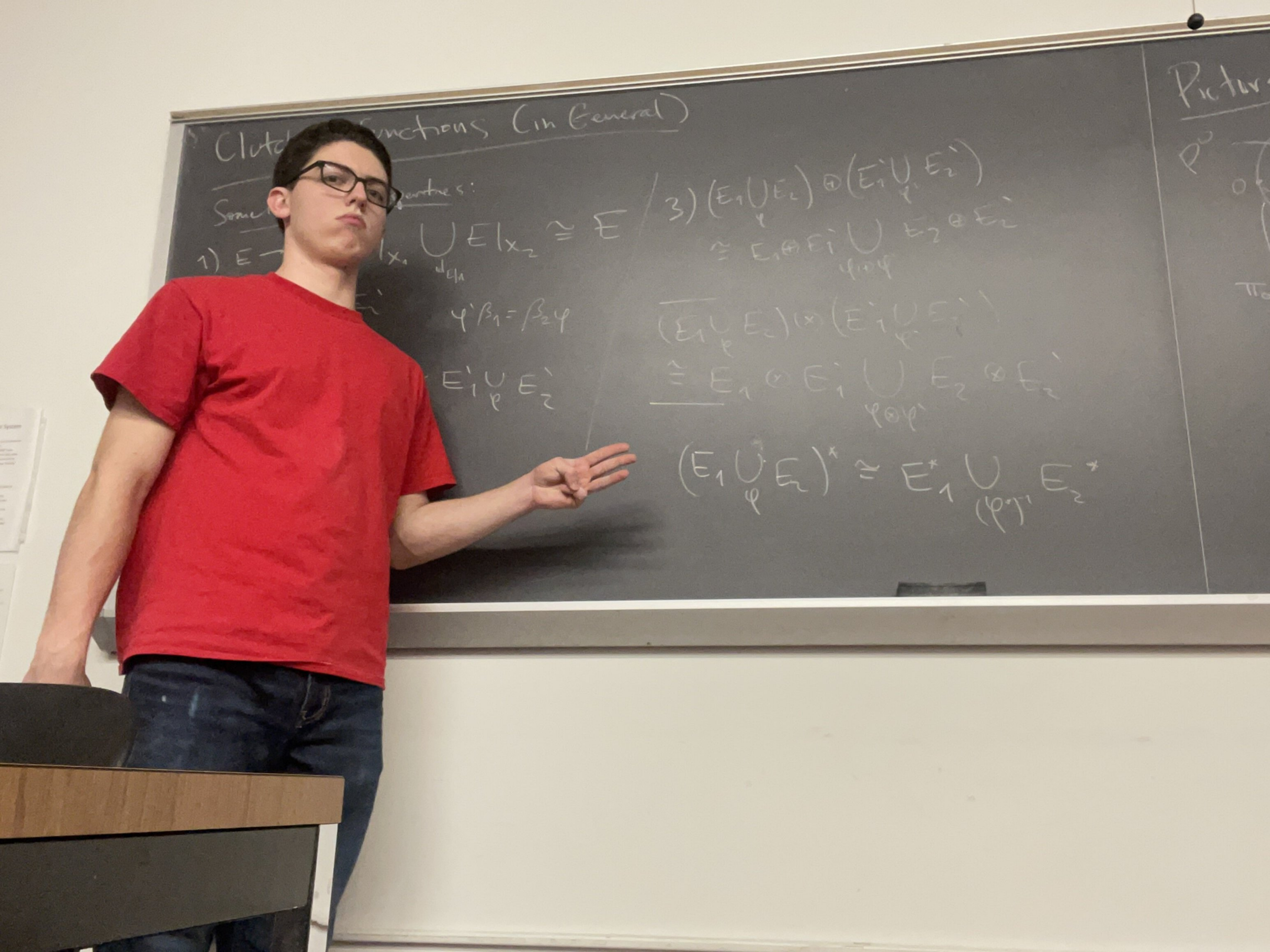

(in General)
1) E → X
E|X2 ≅ E
φ'β1 = β2φ
E'1 ∪φ E'2
3) (E1 ∪φ E2) ⊕ (E'1 ∪φ' E'2)
≅ E1 ⊕ E'1 ∪φ⊕φ' E2 ⊕ E'2
(E1 ∪φ E2) ⊗ (E'1 ∪φ' E'2)
≅ E1 ⊗ E'1 ∪φ⊗φ' E2 ⊗ E'2


Clutch Functions (in General)
Some properties:
1) E → E|X1 ∪ E|X2 ≅ E
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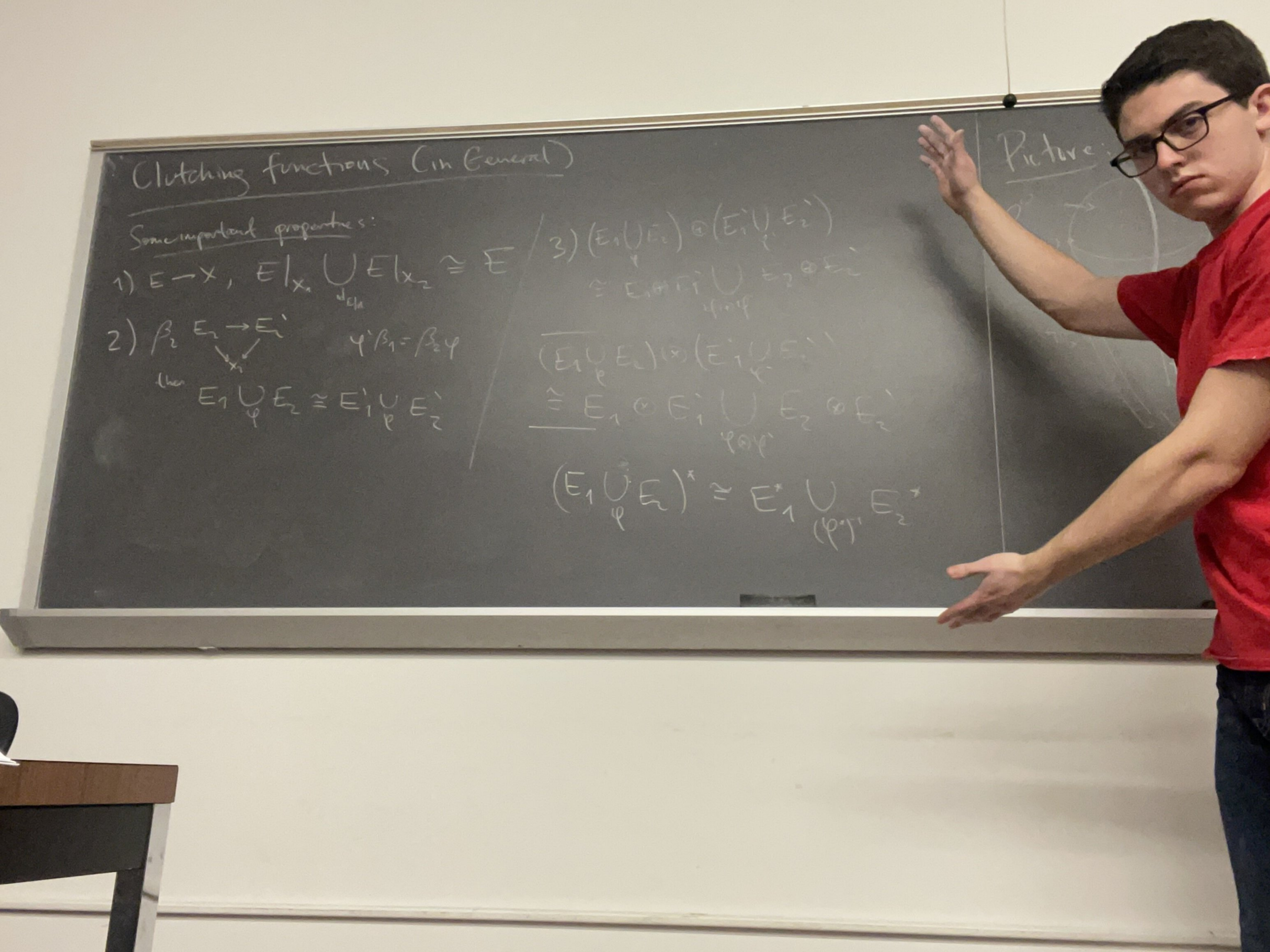
E'1 ∪φ E'2
3) (E1 ∪φ E2) ⊕ (E'1 ∪φ' E'2)
≅ E1 ⊕ E'1 ∪φ⊕φ' E2 ⊕ E'2
(E1 ∪φ E2) ⊗ (E'1 ∪φ' E'2)
≅ E1 ⊗ E'1 ∪φ⊗φ' E2 ⊗ E'2
(E1 ∪φ E2)* ≅ E*1 ∪(φ*)⁻¹ E*2
Pictur


Clutching functions (in General)
Some important properties:
1) $E \to X$, $E|_{X_1} \cup_{id_{E|_A}} E|_{X_2} \cong E$
2) $\beta_2$ $E_2 \to E_2'$ $\varphi' \beta_1 = \beta_2 \varphi$
$X_1$
then $E_1 \cup_\varphi E_2 \cong E_1' \cup_\varphi E_2'$
3) $(E_1 \cup_\varphi E_2) \oplus (E_1' \cup_{\varphi'} E_2')$
$\cong E_1 \oplus E_1' \cup_{\varphi \oplus \varphi'} E_2 \oplus E_2'$
$(E_1 \cup_\varphi E_2) \otimes (E_1' \cup_{\varphi'} E_2')$
$\cong E_1 \otimes E_1' \cup_{\varphi \otimes \varphi'} E_2 \otimes E_2'$
$(E_1 \cup_\varphi E_2)^* \cong E_1^* \cup_{(\varphi^*)^{-1}} E_2^*$
Picture:

Clutching functions (in General)

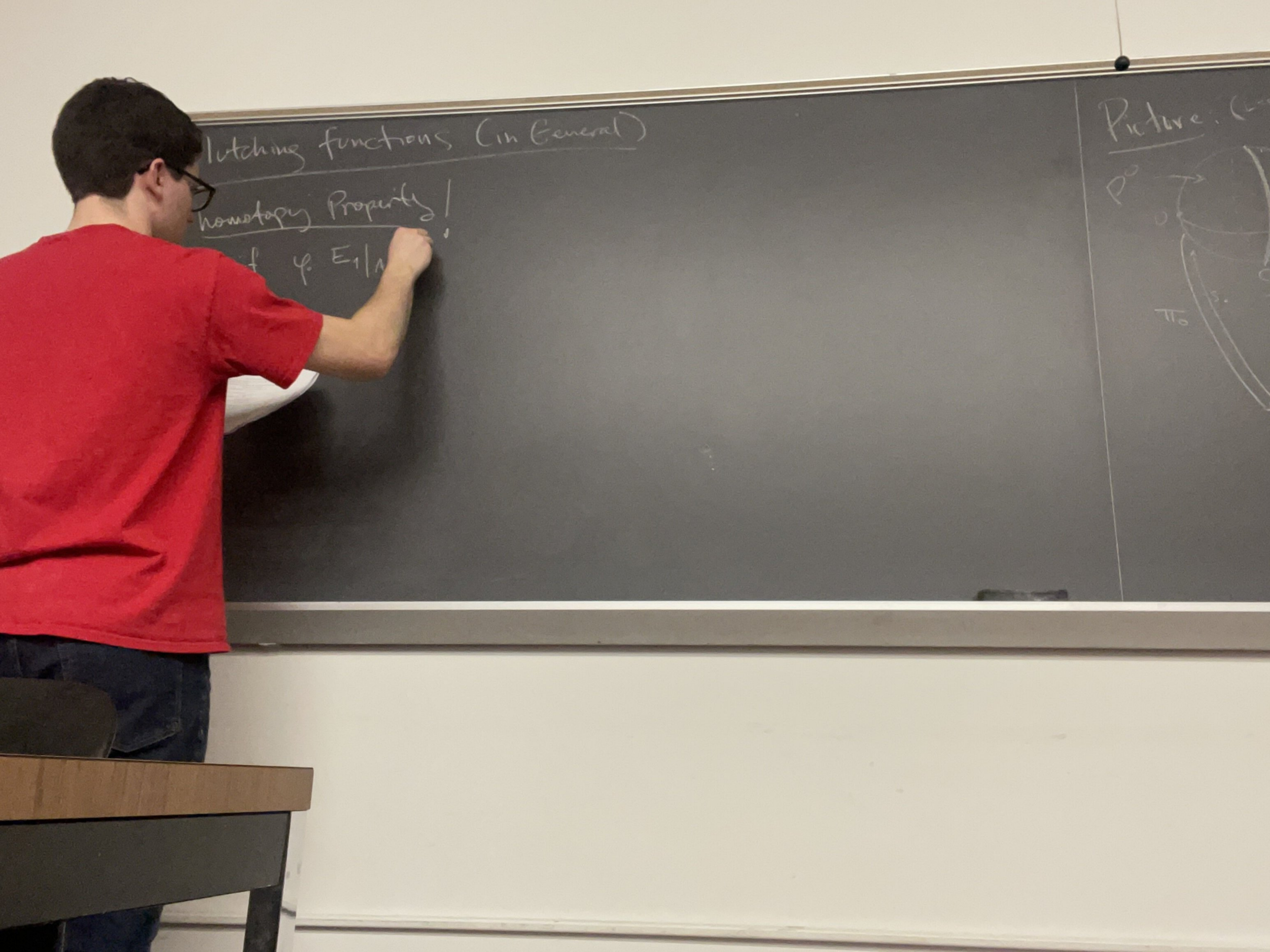
Homotopy Property

Picture:

Clutching functions (in General)

homotopy Property!

Picture:


lutching functions (in General)
homotopy Property!
Picture:

# Clutching functions (in General)

## homotopy Property

**Lemma** if $\varphi_0 : E_1|_A \xrightarrow{\cong} E_2|_A$
and $\varphi_1 : E_1|_A \xrightarrow{\cong} E_2|_A$
are homotopic as VB isos

then

Picture: (

# Clutching functions (in General)

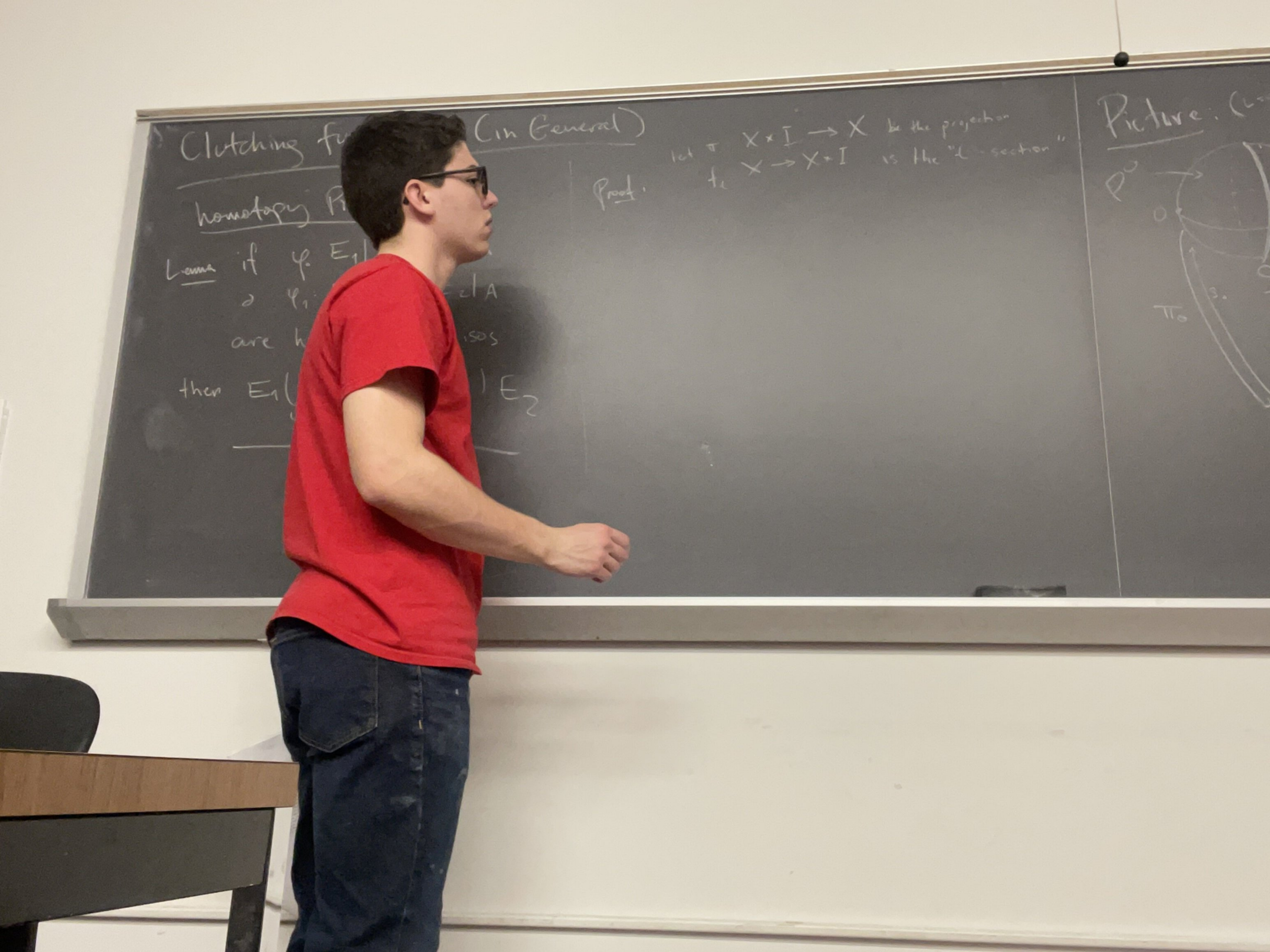
## homotopy Property!

**Lemma** if $\varphi_0 : E_1|_A \xrightarrow{\cong} E_2|_A$
and $\varphi_1 : E_1|_A \xrightarrow{\cong} E_2|_A$
are homotopic as VB isos

then $E_1 \bigcup_{\varphi_0} E_2 \cong E_1 \bigcup_{\varphi_1} E_2$

Picture: (


(in General)
Proof:
let π X × I → X be the projection
Picture:


Clutching fu
(in General)
homotopy Pr
Lemma if φ₀ E₁|
are h
isos
then E₁(
) E₂
Proof:
let π X × I → X be the projection
t₀ X → X × I is the "t-section"
Picture:
P^0
π₀

Clutching functions

homotopy Property

Lemma if $\varphi: E_1|_A \xrightarrow{\sim} E'$

$\varphi_1: E_1|_A \xrightarrow{\sim}$

are homotop

then $E_1 \bigcup_{\varphi} E_2$

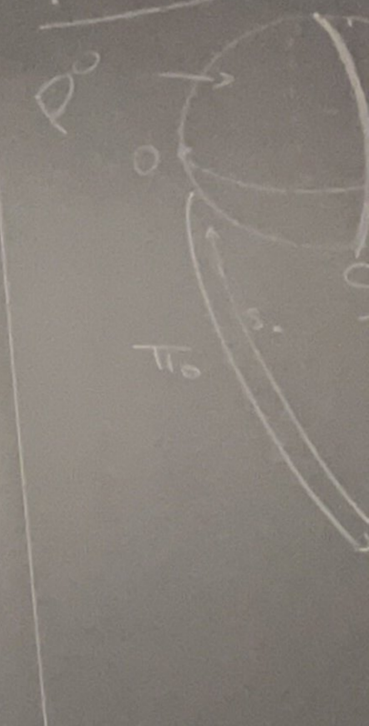
Proof.

let $\pi: X \times I \to X$ be the projection

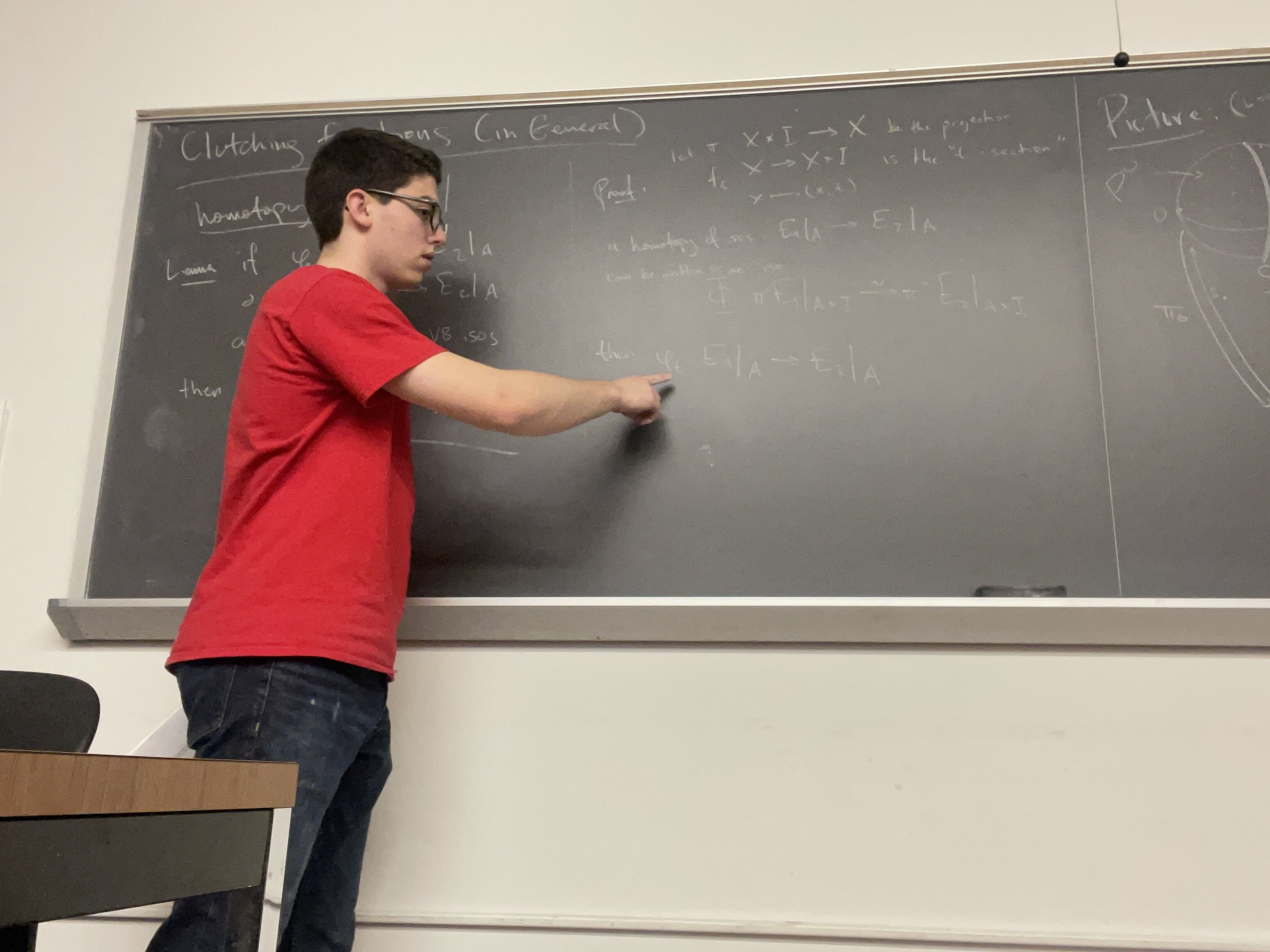
$f_t: X \to X \times I$ is the "$t$" section"

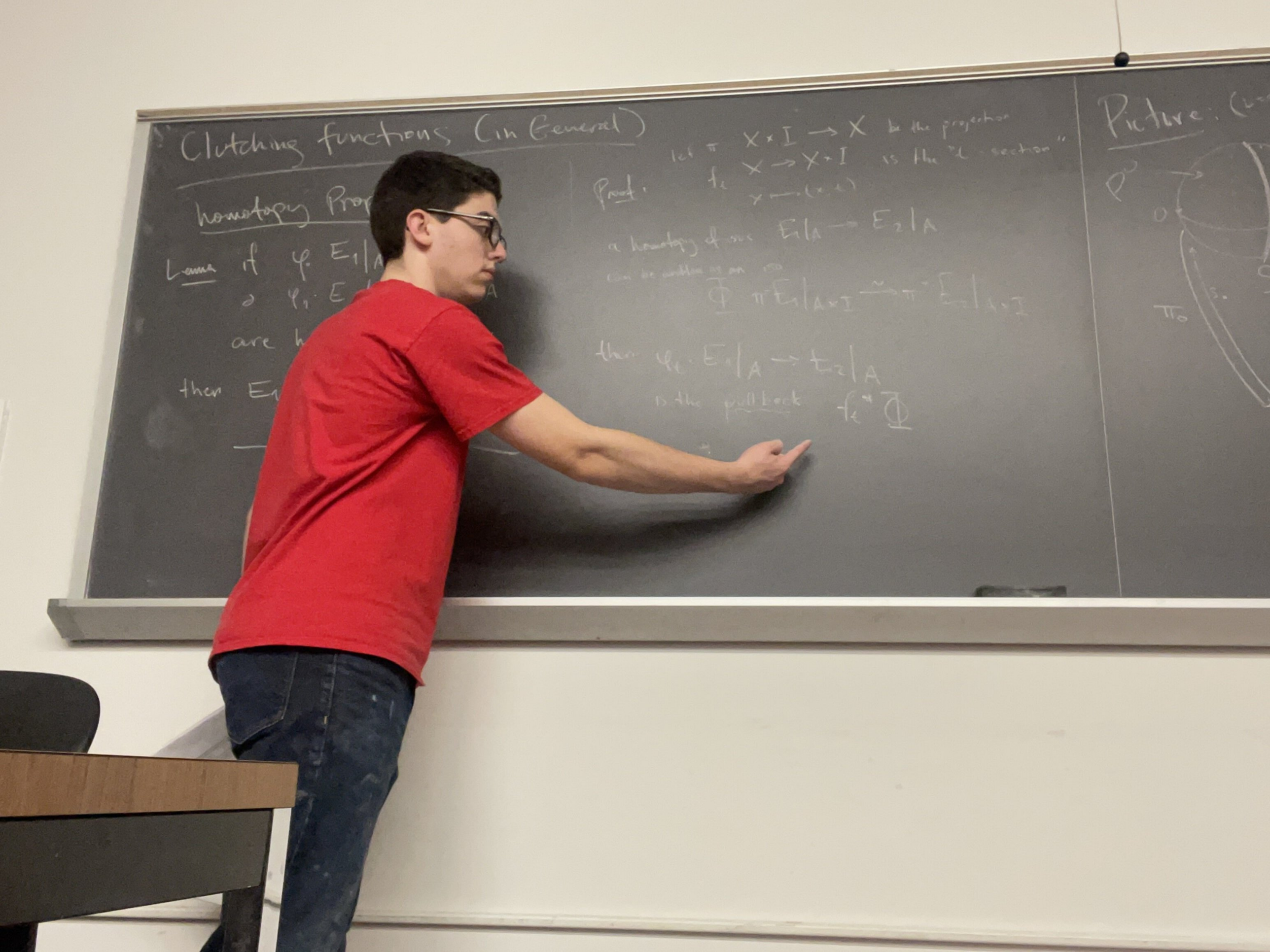
$x \mapsto (x,$

Picture:




Clutching functions (in General)
homotopy
Lemma
Proof:
let π X × I → X be the projection
f_t X → X × I is the "t-section"
x ↦ (x, t)
a homotopy of isos E_1|_A → E_2|_A
Picture:


Clutching
(in General)
homotopy
Lemma if
then
Proof:
let π X × I → X be the projection
f_t X → X × I is the "t-section"
x ↦ (x, t)
a homotopy of isos E1|A → E2|A
can be written as an iso
Φ π*E1|A×I ≃ π*E2|A×I
then φ_t E1|A → E2|A
Picture:
π0


Clutching functions (in General)
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Lemma if
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f_t X → X × I is the "t-section"
x ↦ (x, t)
a homotopy of isos E_1|_A → E_2|_A
can be written as an iso
Φ π^* E_1|_{A×I} ≃ π^* E_2|_{A×I}
then φ_t : E_1|_A → E_2|_A
is the pullback f_t^* Φ
Picture:
π_0

# Clutching functions (in General)

## homotopy Property I

**Lemma** if $\varphi_0 : E_1|_A \xrightarrow{\cong} E_2|_A$
and $\varphi_1 : E_1|_A \xrightarrow{\cong} E_2|_A$
are homotopic as VB isos

then $E_1 \underset{\varphi_0}{\cup} E_2 \cong E_1 \oplus E_2$

---

Let $\pi : X \times I \to X$ be the projection
$i_t : X \to X \times I$ is the "$t$-section"
$x \longmapsto (x, t)$

**Prop.** is a homotopy of isos $E_1|_A \to E_2|_A$
can be written as an iso

$$\Phi : \pi^* E_1|_{A \times I} \xrightarrow{\sim} \pi^* E_2|_{A \times I}$$

then if $E_1|_A \to E_2|_A$
is the pullback $i_t^* \Phi$

$$E_1 \underset{\varphi_0}{\cup} E_2 \cong i_0^* \left( \pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2 \right)$$

Picture: (

# Clutching functions (in General)

**homotopy Property!**

**Lemma** if $\varphi_0 : E_1|_A \xrightarrow{\cong} E_2|_A$
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then $E_1 \underset{\varphi_0}{\cup} E_2 \cong E_1 \amalg E_2$

---

Let $\pi : X \times I \to X$ be the projection
$f_t : X \to X \times I$ is the "$t$-section"
$x \mapsto (x, t)$

**Proof:** a homotopy of iso $E_1|_A \to E_2|_A$
can be written as an iso

$$\Phi : \pi^* E_1|_{A\times I} \xrightarrow{\cong} \pi^* E_2|_{A\times I}$$

then $\varphi_t : E_1|_A \to E_2|_A$
is the pullback $f_t^* \Phi$

$$E_1 \underset{\varphi_0}{\cup} E_2 \cong f_0^*\left(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2\right)$$

$$E_1 \underset{\varphi_1}{\cup} E_2 \cong f_1^*\left(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2\right)$$

**Picture:** (

# Clutching functions (in General)

**homotopy Property**

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then $E_1 \underset{\varphi_0}{\cup} E_2 \cong E_1 \oplus E_2$

---

Proof. let $\pi : X \times I \to X$ be the projection
let $i_t : X \to X \times I$ is the "$t$ section"
$x \longmapsto (x,t)$

a homotopy of isos $E_1|_A \longrightarrow E_2|_A$
can be written as an iso

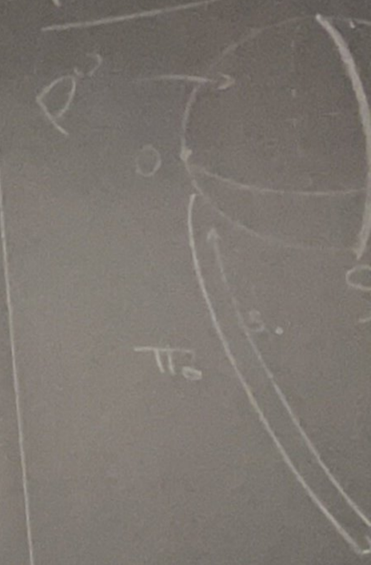
$$\Phi : \pi^* E_1|_{A\times I} \xrightarrow{\cong} \pi^* E_2|_{A\times I}$$

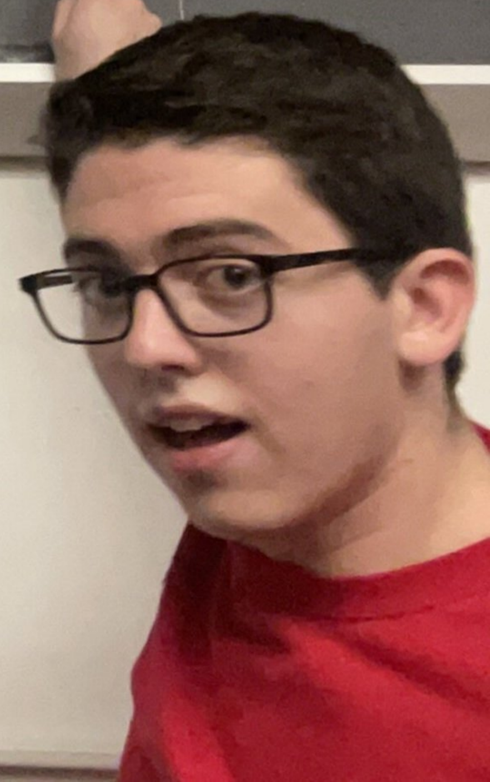
then $\varphi_t : E_1|_A \to E_2|_A$
is the pull back $i_t^* \Phi$

$$E_1 \underset{\varphi_0}{\cup} E_2 \cong i_0^*(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2)$$

$$E_1 \underset{\varphi_1}{\cup} E_2 \cong i_1^*(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2)$$

Picture:



# Clutching functions (in General)

## homotopy Property!

**Lemma** if $\varphi_0 : E_1|_A \xrightarrow{\cong} E_2|_A$
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then $E_1 \underset{\varphi_0}{\cup} E_2 \cong E_1 \oplus E_2$

---

Let $\pi : X \times I \to X$ be the projection
$f_t : X \to X \times I$ is the "$t$ section"
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**Proof.** a homotopy of isos $E_1|_A \to E_2|_A$
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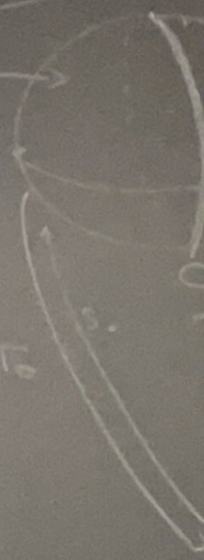
then if $E_1|_A \to E_2|_A$
is the pullback $f_t^* \Phi$

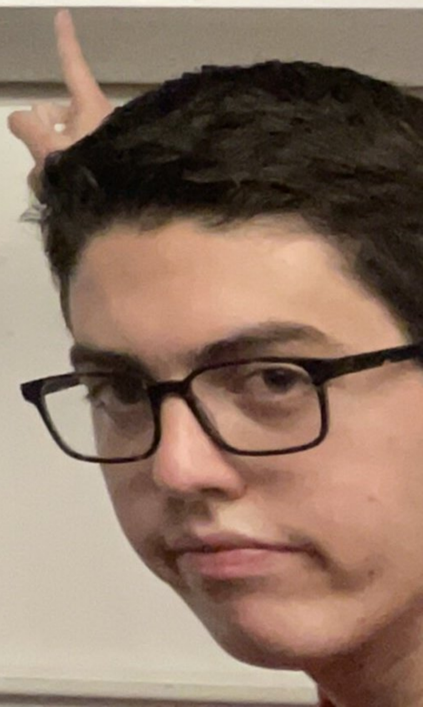
$$E_1 \underset{\varphi_0}{\cup} E_2 \cong f_0^*\left(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2\right) \Big\rangle 2$$

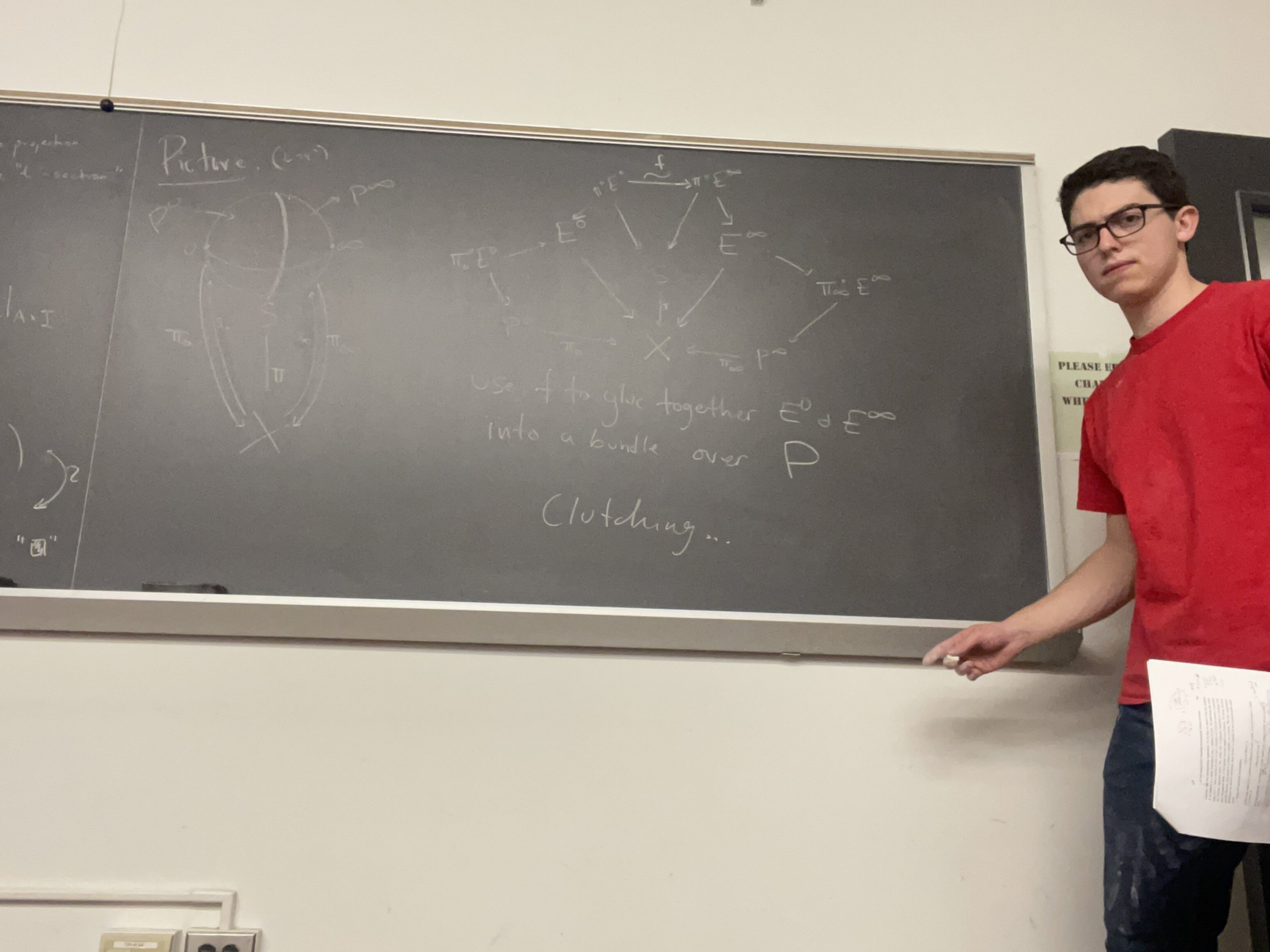
$$E_1 \underset{\varphi_1}{\cup} E_2 \cong f_1^*\left(\pi^* E_1 \underset{\Phi}{\cup} \pi^* E_2\right)$$

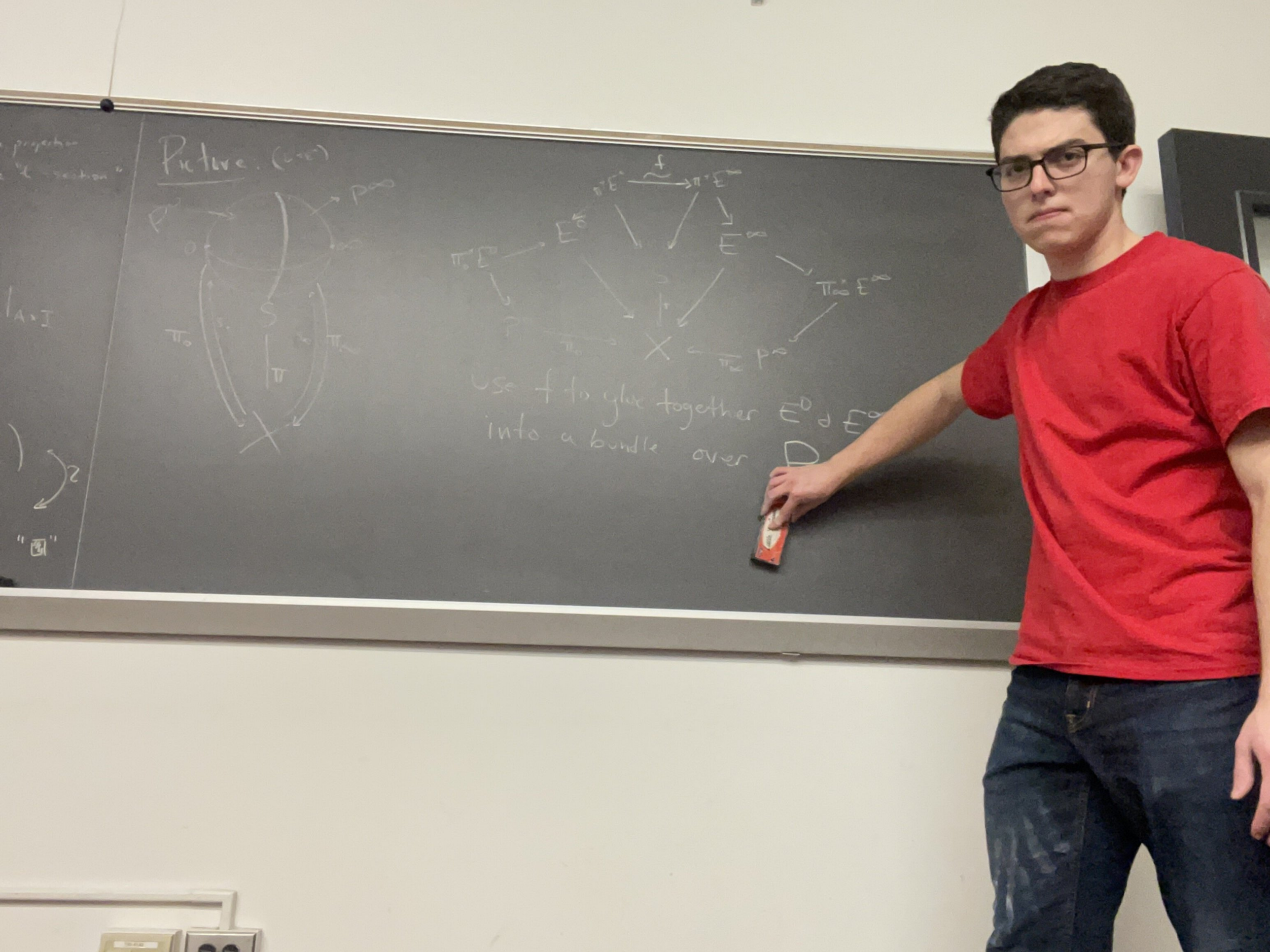
"$\Phi$"

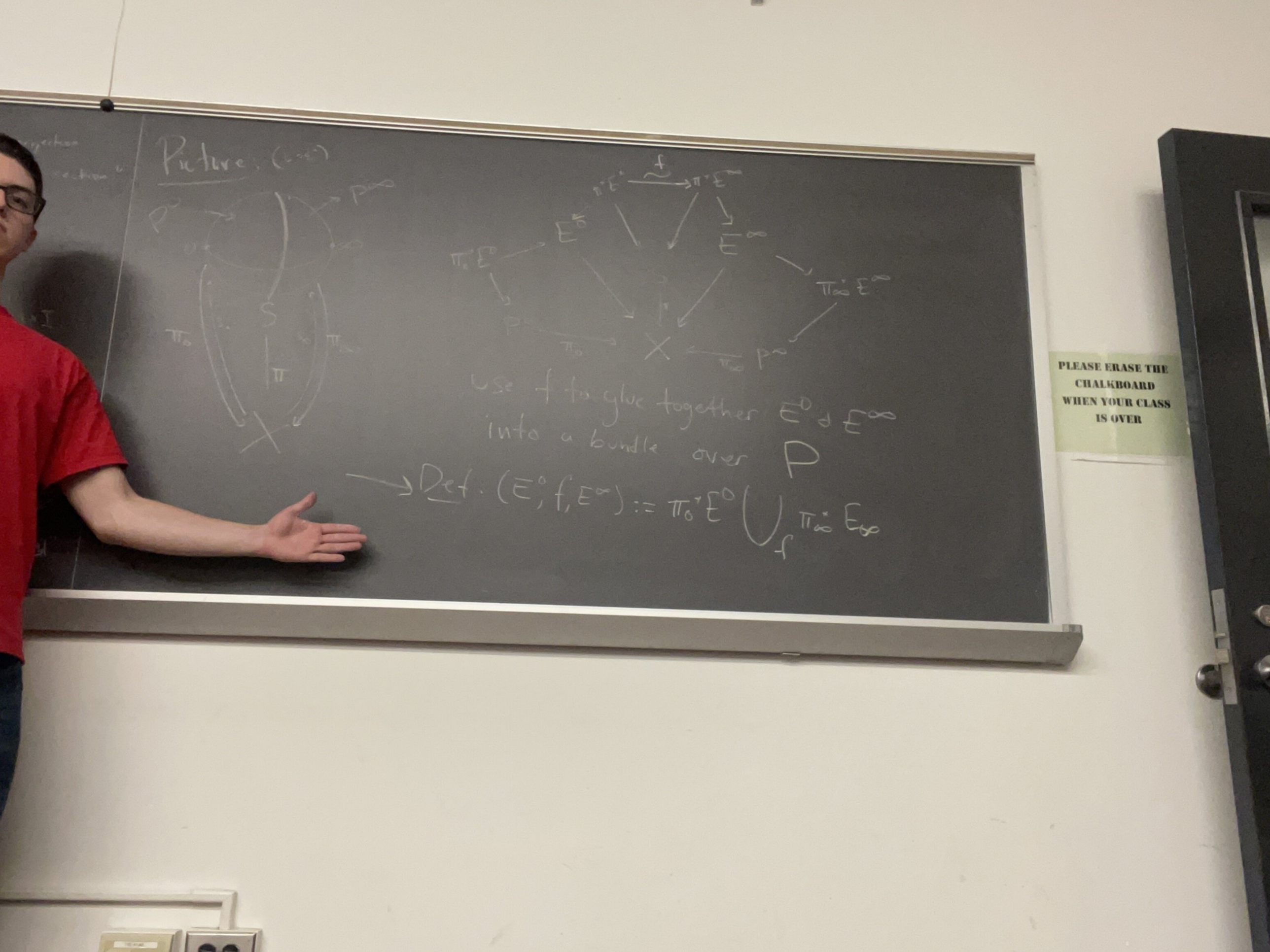
Picture:




Picture.
$\pi^*E^0 \xrightarrow{f} \pi^*E^\infty$
$E^0$
$E^\infty$
$\pi_0^*E^0$
$\pi_\infty^*E^\infty$
$P^0$
$P^\infty$
$X$
$\pi_0$
$\pi_\infty$
use $f$ to glue together $E^0$ & $E^\infty$
into a bundle over $P$
Clutching...


Picture.
Use f to glue together E^0 & E^∞
into a bundle over D


Picture: (...)
$P^0$
$P^\infty$
$S$
$\pi$
$\pi_0$
$\pi_\infty$
$X$
$\pi^*E^0$
$\pi^*E^\infty$
$E^0$
$E^\infty$
$\pi_0^*E^0$
$\pi_\infty^*E^\infty$
$P$
$X$
$P^\infty$
use $f$ to glue together $E^0$ & $E^\infty$
into a bundle over $P$
Def. $(E^0, f, E^\infty) := \pi_0^*E^0 \bigcup_f \pi_\infty^* E_\infty$
PLEASE ERASE THE CHALKBOARD WHEN YOUR CLASS IS OVER


Clutching functions (in Pcase)
Lemma

Clutching functions (in P case)

Lemma $E \to P$ any bundle

# Clutching functions (in P case)

**Lemma** $E \to P$ any bundle

$E^0 = s_0^* E \to X$

& $E^\infty = s_\infty^* E \to X$

# Clutching functions (in P case)

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# Clutching functions (in P case)

**Lemma** $E \to P$ any bundle

$E^0 = s_0^* E \to X$

& $E^\infty = s_\infty^* E \to X$

$\exists\ f\ \pi^* E^0 \xrightarrow{\sim} \pi^* E^\infty$

st $E \cong (E^0, f, E^\infty)$

# Clutching functions (in P case)

**Lemma** $E \to P$ any bundle

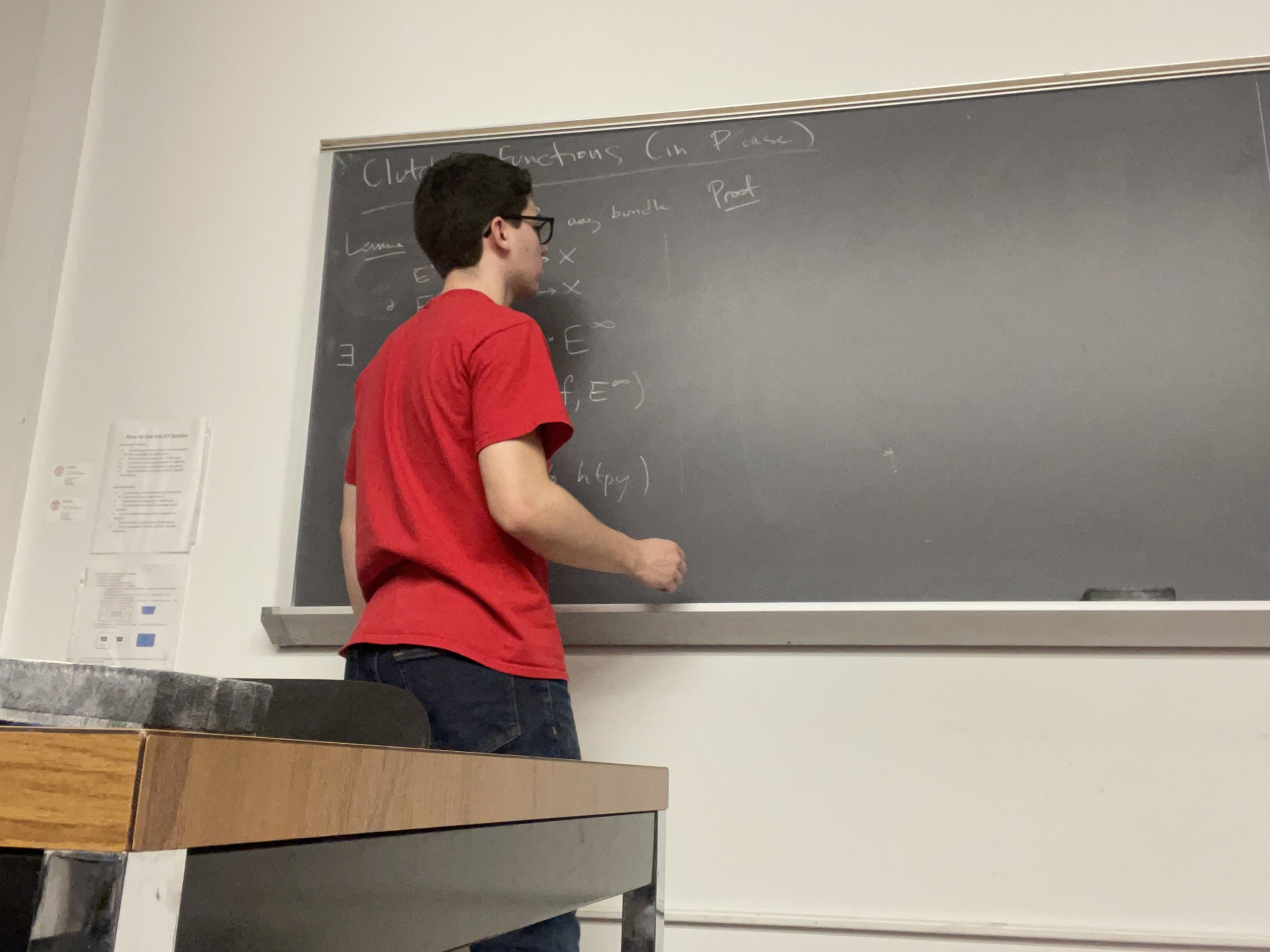
$E^0 = s_0^* E \to X$

& $E^\infty = s_\infty E \to X$

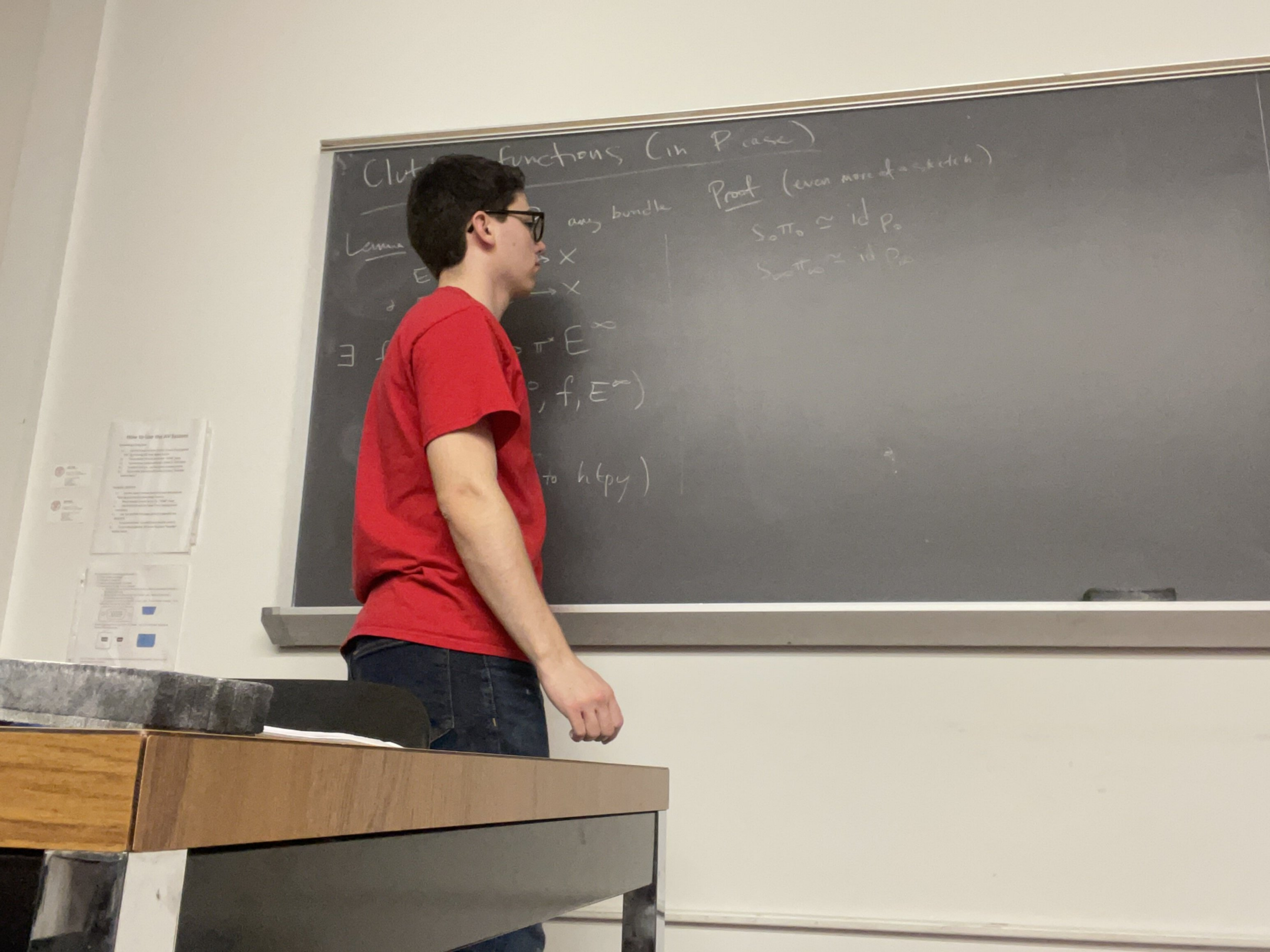
$\exists\, f\ \pi^* E^0 \xrightarrow{\sim} \pi^* E^\infty$

st. $E \cong (E^0, f, E^\infty)$

(f unique up to htpy)


functions (in P case)
any bundle
Proof
Lemma
htpy)


Clutching functions (in Pease)
Lemma
Proof (even more of a sketch)


functions (in P case)
any bundle
Proof (even more of a sketch)
$S_0\pi_0 \simeq id_{P_0}$
$S_\infty\pi_\infty \simeq id_{P_\infty}$
$\pi^* E^\infty$
$f, E^\infty)$
to htpy)

# Clutching functions (in P case)

**Lemma** $E \to P$ any bundle

$E^0 := s_0^* E \to X$

$E^\infty := s_\infty^* E \to X$

$\exists\, f\ \pi^* E^0 \xrightarrow{\sim} \pi^* E^\infty$

s.t. $E \cong (E^0, f, E^\infty)$

(f unique up to htpy)

**Proof** (even more sketchy)

$s_0 \pi_0 \simeq id_{P_0} \rightsquigarrow E|_{P_0} \cong \pi_0^* s_0^* E = \pi_0^* E^0$

$s_\infty \pi_\infty \simeq id_{P_\infty} \rightsquigarrow E|_{P_\infty} \cong \pi_\infty^* s_\infty^* E = \pi_\infty^* E^\infty$

Clutching f[illegible]s (in P case)

Lemma $E \to$ [illegible]

$E^D \simeq s_0^* E$

$\exists f$

st

Proof (even more abstract)

$s_0 \pi_0 \simeq id_{P_0} \rightsquigarrow E|_{P_0} \xrightarrow{f_0} \pi_0^* s_0^* E = \pi_0^* E^D$

$s_\infty \pi_\infty \simeq id_{P_\infty} \rightsquigarrow E|_{P_\infty} \xrightarrow{f_\infty} \pi_\infty^* s_\infty^* E = \pi_\infty^* E^D$

choose there in the right way to make $f_0 \circ f_\infty = id|_S$

Picture: (C[illegible]

$P^D$

$\pi_0$

Clutching functions (in P

Lemma $E \to P$ any bundle

$E^\circ := s_0^* E \to X$

& $E^\infty := s_\infty^* E \to X$

$\exists\, f : \pi^* E^\circ \xrightarrow{\sim} \pi^* E^\infty$

st $E \cong (E^\circ, f, E^\infty)$

(f unique up to htpy)

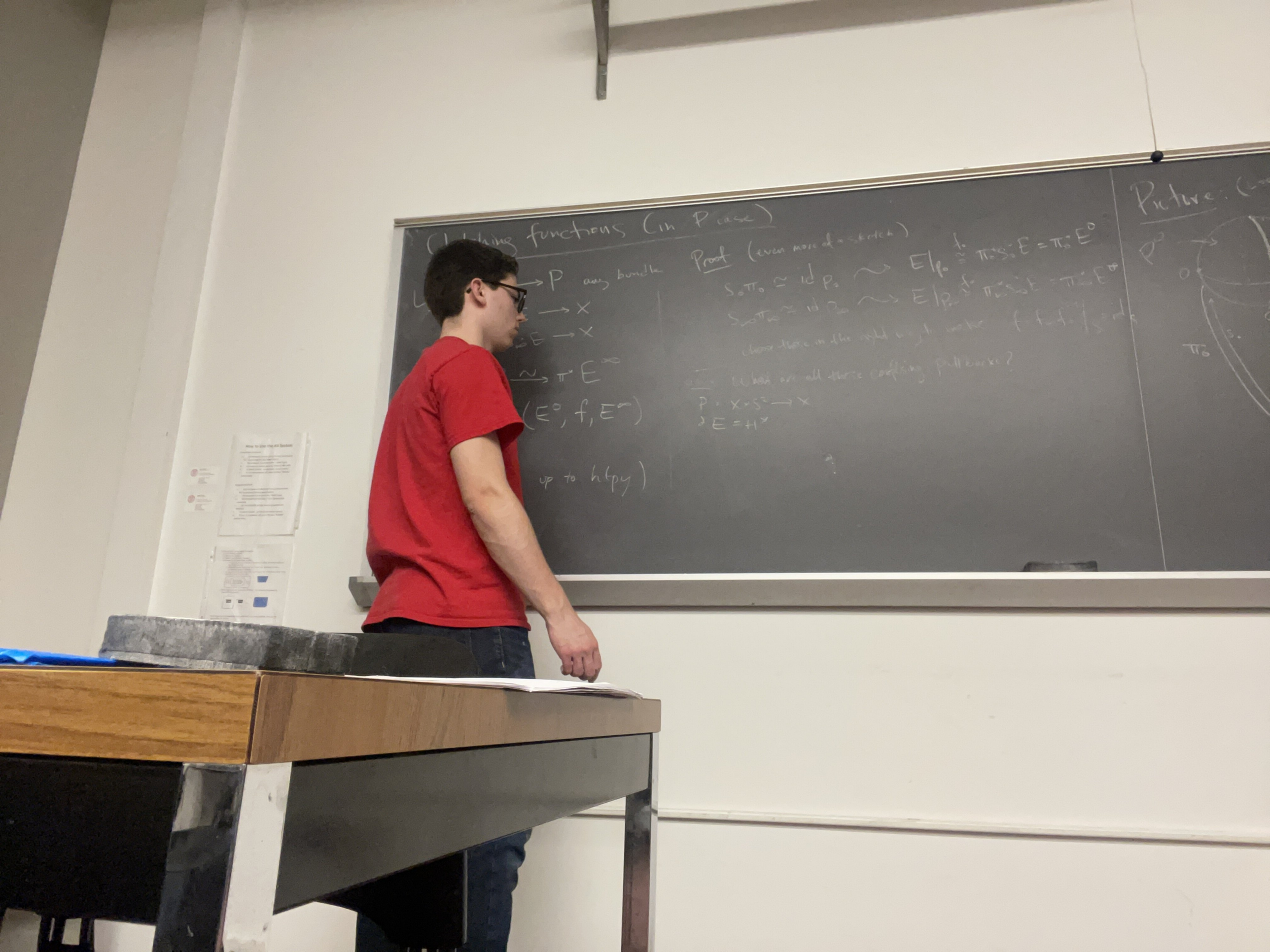
Picture

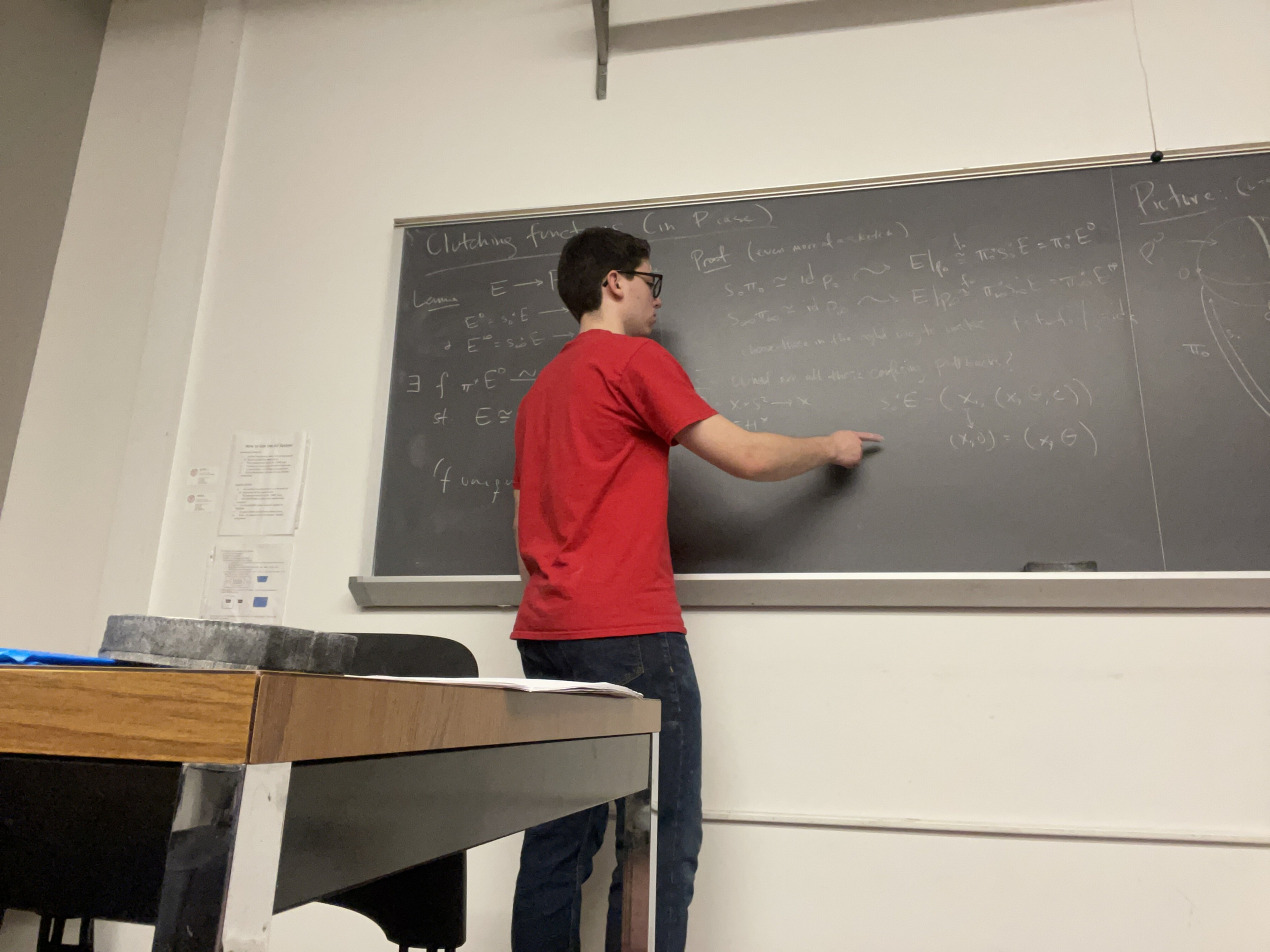
Clutching functions (in P case)

Lemma

Proof

Picture


functions (in P case)
Proof (even more of a sketch)
any bundle
up to htpy)
Picture:


Clutching functions (in P case)
Lemma
Proof (even more of a sketch)
Picture:

Clutching functions (in P

Lemma $E \to P$ any bundle

$E^0 = s_0^* E \to X$

& $E^\infty = s_\infty^* E \to X$

$\exists\, f\ \pi^* E^0 \xrightarrow{\sim} \pi^* E^\infty$

st. $E \cong (E^0, f, E$

(f unique up to h

id $p_0 \sim E|_{p_0}$ $\overset{t_0}{\cong}$ $\pi_0^* s_0^* E = \pi_0^* E^0$

id $p_\infty \sim E|_{p_\infty}$ $\overset{t_\infty}{\cong}$ $\pi_\infty^* s_\infty^* E = \pi_\infty^* E^\infty$

$s_0^* E = (X_1, (X_2, \theta, c))$

$(X_1, 0) = (X_2, \theta)$

Picture

$P^1$

$\pi_0$

Clu ctions (in P case)

Lemma

any bundle

Proof (seven more attempts)

$S_0 \pi_0 \simeq id\ P_0 \sim E|_{P_0}$

$\pi_0^* S_0^* E = \pi_0^* E^D$

$S_\infty \pi_\infty \simeq id\ P_\infty \sim E|_{P_\infty}$

$\pi_\infty^* S_\infty^* E = \pi_\infty^* E^\infty$

(hmm there is the right way to make $\{ t_\alpha f_\alpha \}$ ...)

(what are all these coefficient problems?)

$P = X \vee S^2 \to X$

$\partial E = t 1^\vee$

$S_0^* E = (X_1, (X_1, \theta, c))$

$(X_1, 0) = (X_1, \theta)$

$\pi_0^* S_0^* E = ((X_1, \theta_2), (X_2, 0, c))$

$X_1 = X_2$

Picture

$P^D$

$\pi_0$


Clutching functions (in P case)
Lemma
Picture:

Clutching functions (in P case)

Lemma $E \to P$ any bundle

$E^\circ = s_0^* E \to X$

$E^\infty = s_\infty^* E \to X$

$\exists\, f : \pi^* E^\circ \xrightarrow{\sim} \pi^* E^\infty$

s.t. $E \cong (E^\circ, f, E^\infty)$

(f unique up to htpy)

Proof

Picture

Clutching functions

Lemma $E \to P$ any bun

$E^\circ = s_0^* E \to X$

$E^\bullet = s_\infty^* E \to X$

$\exists$ ! $\pi^* E^\circ \xrightarrow{\sim} \pi^*$

s.t. $E \cong (E^\circ,$

(even more at a stretch)

$s_0 \pi_0 \simeq id_{P_0}$ $\rightsquigarrow$ $E|_{P_0} \cong \pi_0^* s_0^* E = \pi_0^* E^\circ$

$s_\infty \pi_\infty \simeq id_{P_\infty}$ $\rightsquigarrow$ $E|_{P_\infty} \cong \pi_\infty^* s_\infty^* E = \pi_\infty^* E^\bullet$

Where there is the will there is a way to make $f$ [illegible]

What are all these conditions? It works?

$X \sim S^2 \to X$

$= +1^\vee$

$s_0^* E = ((X_1, (X_1, \theta_1, c))$

$(X_1, 0) = (X_1, \theta)$

$f$ unique up t

$s_0^* s_0 E = ((X_1, \theta_2), (X_2, 0, c))$

$X_1 = X_2$

Picture. (

$P^1$

$\pi_0$

Bad picture


We need to understand
Clutching functions up to htpy

We need to understand
Clutching functions up to htpy

$$\begin{array}{ccc} \pi^*L & \longrightarrow & L \\ \downarrow & & \downarrow \\ S & \longrightarrow & X \end{array}$$

Bad picture

Please)

Proof (even more of a sketch)

$s_0\pi_0 \simeq id_{p_0} \rightsquigarrow E|_{p_0} \overset{f_0}{\cong} \pi_0^* s_0^* E = \pi_0^* E^0$

$s_\infty\pi_\infty \simeq id_{p_\infty} \rightsquigarrow E|_{p_\infty} \overset{f_\infty}{\cong} \pi_\infty^* s_\infty^* E = \pi_\infty^* E^\infty$

choose these in the right way to make $f = f_\infty f_0^{-1}|_S = id_E$

aside What are all these confusing pullbacks?

$P = X \times S^2 \to X$

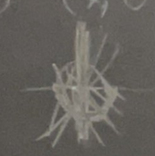
$\partial E = H^*$

$s_0^* E = (x_1, (x_1, \theta, c))$

$(x_1, 0) = (x_1, \theta)$

$\pi_0^* s_0^* E = ((x_1, \theta_2), (x_2, 0, c))$

$x_1 = x_2$



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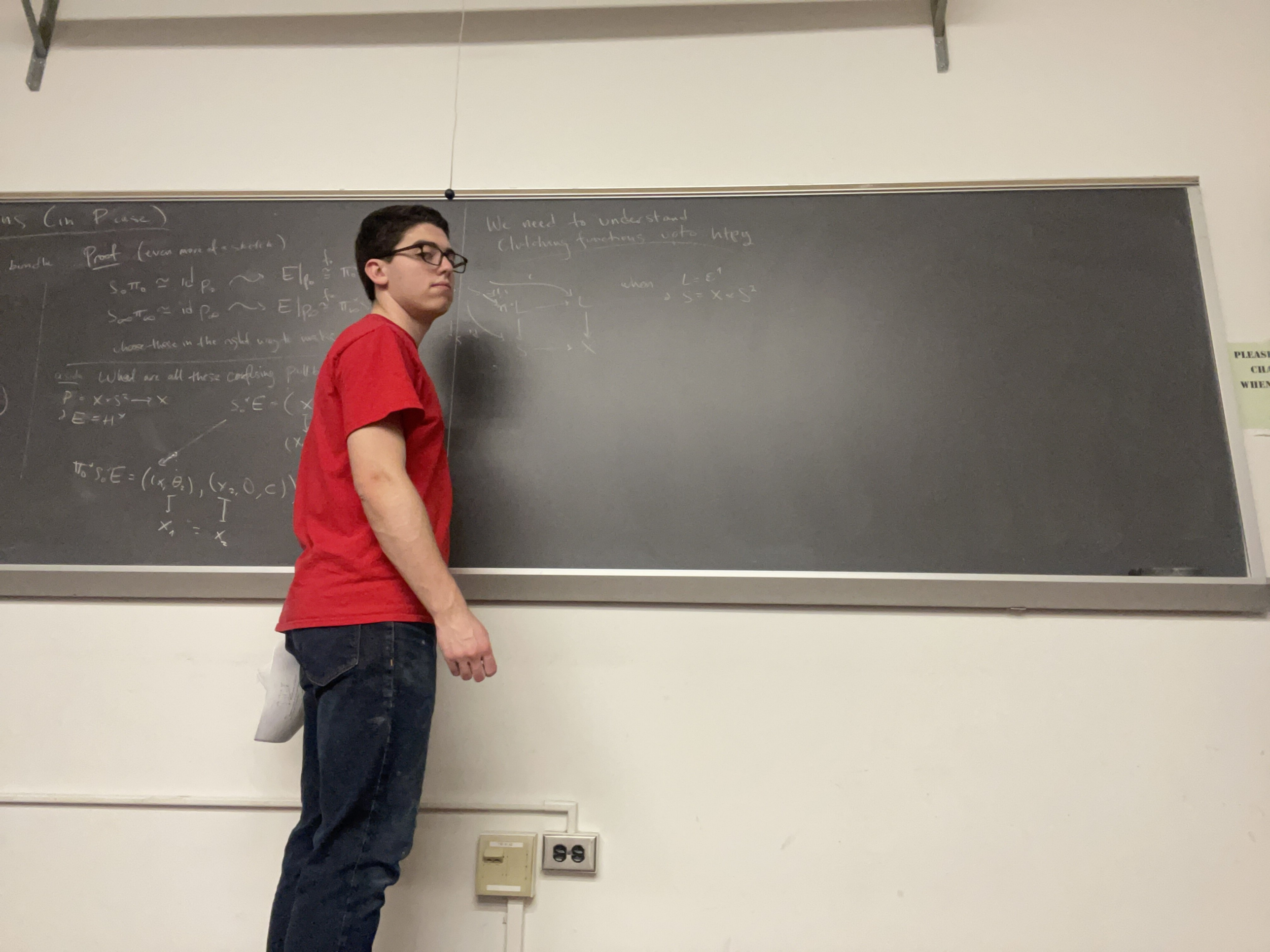
We need to understand clutching functions upto htpy

$$\begin{tikzcd} S \arrow[r, bend left, "i"] \arrow[dr, bend right] & L \arrow[d] \\ & X \end{tikzcd}$$

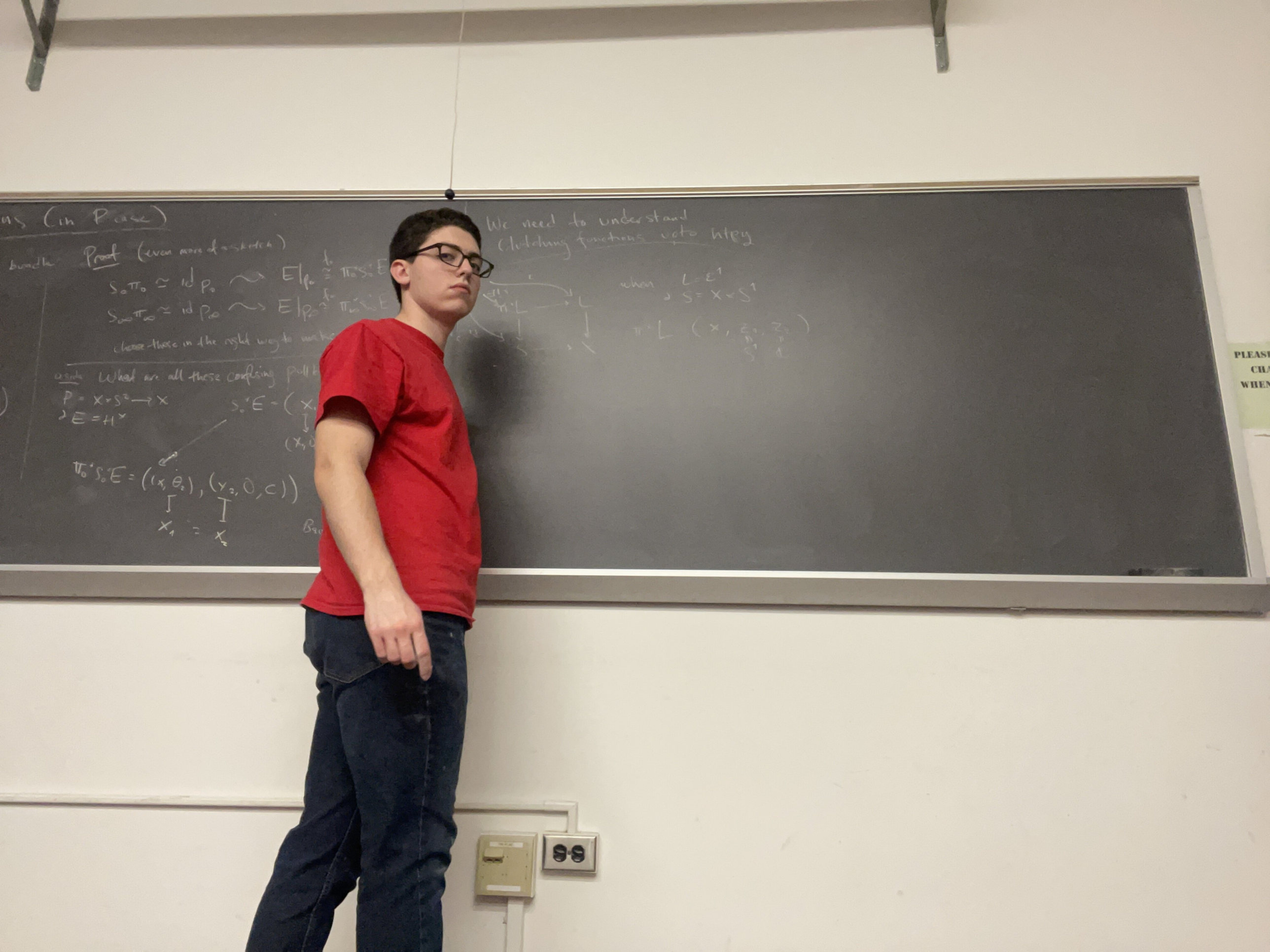
$$\begin{tikzcd} \pi^* L \arrow[r] \arrow[d] & L \arrow[d] \\ S \arrow[r] & X \end{tikzcd}$$



Proof (even more of a sketch)
We need to understand
Clutching functions upto htpy



(in P case)
Proof (even more of a sketch)
We need to understand
Clutching functions upto htpy
when L = ε¹
S = X × S²



Proof (even more of a sketch)
We need to understand
clutching functions up to htpy
when L = ε¹
S = X × S¹
What are all these confusing


We need to understand
Clutching functions upto htpy
when L = ε¹
∂ S = X × S¹
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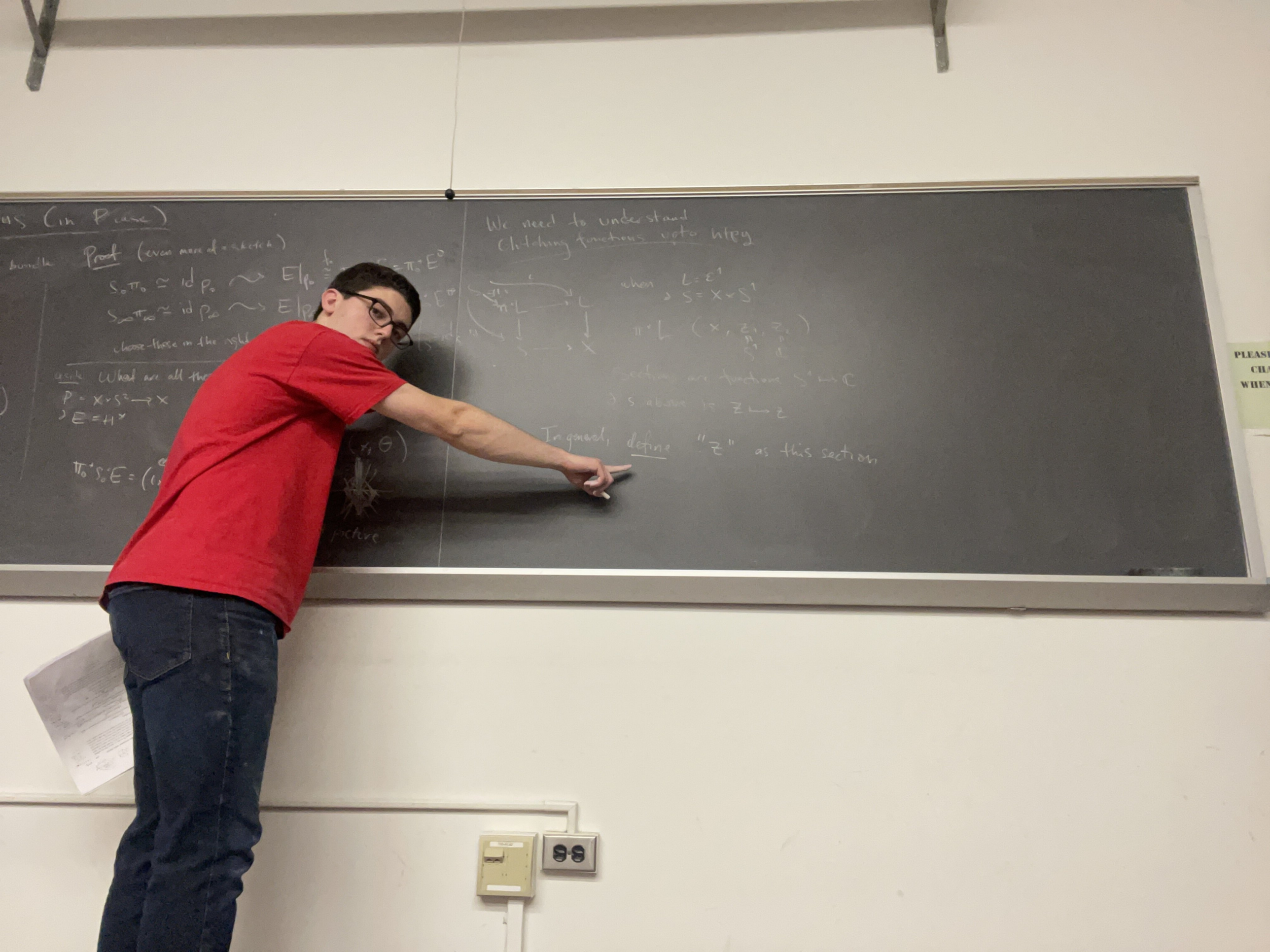
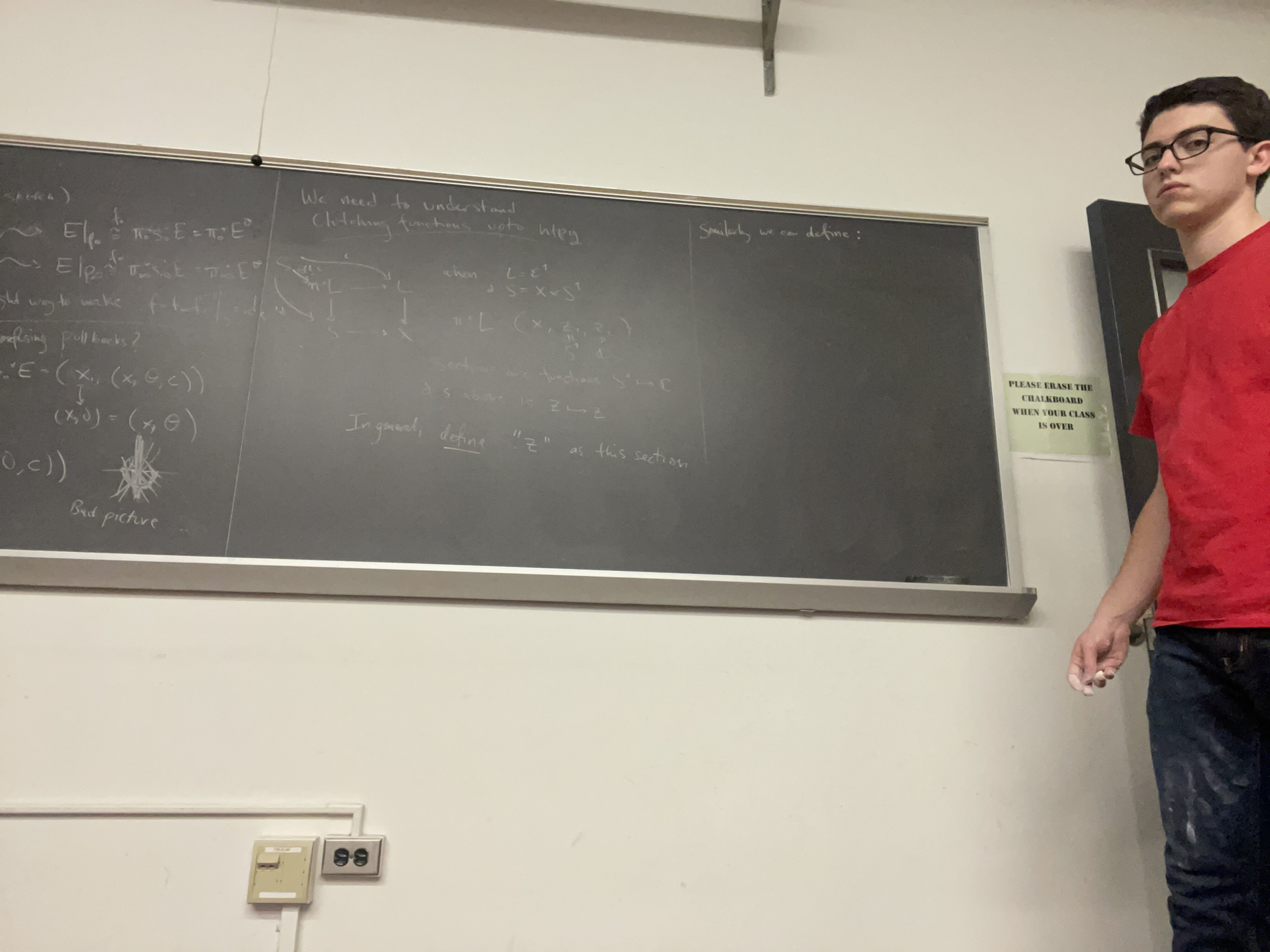


(in Pause)
Proof (even more of a sketch)
We need to understand
clutching functions upto htpy
when L = ε¹
∂ S = X × S¹
In general, define "z" as this section


We need to understand
Clutching functions upto htpy
when L = ε¹
∂ S = X × S¹
In general, define "z" as this section
Similarly we can define :
Bad picture
PLEASE ERASE THE
CHALKBOARD
WHEN YOUR CLASS
IS OVER


We need to understand
Clutching functions upto htpy
when L = ε¹
∂ S = X × S¹
In general, define "z" as this section
Similarly we can define:
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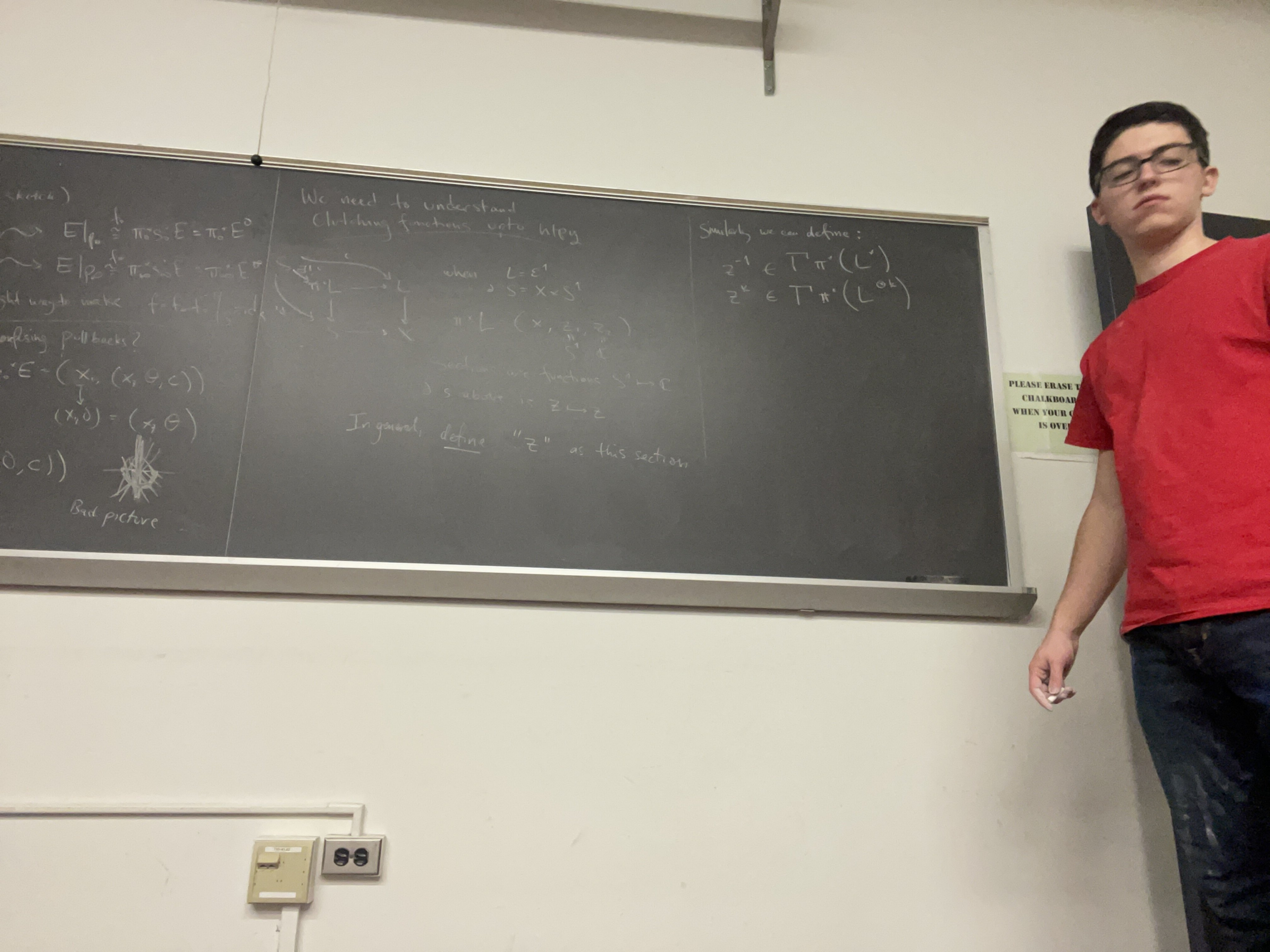
We need to understand
Clutching functions upto htpy
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S = X × S¹
In general, define "z" as this section
Similarly we can define:
Bad picture
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IS OVE

We need to understand
(clutching functions

Similarly we can define:

$$z^{-1} \in \Gamma \pi^* (L^*)$$

$$z^k \in \Gamma \pi^* (L^{\otimes k})$$

for some $a_k \in \Gamma((L^*)^{\otimes k})$

In general,

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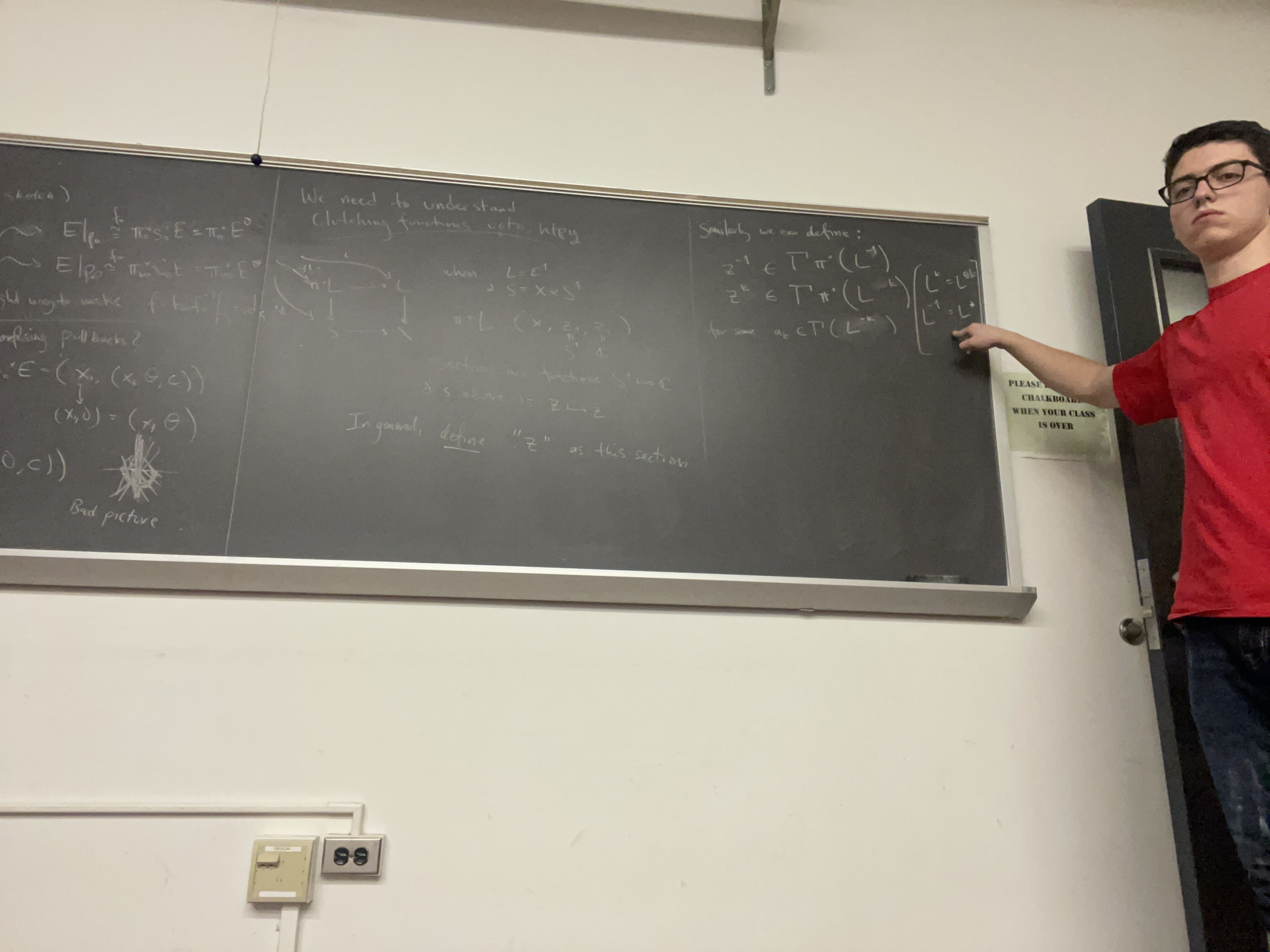

We need to understand
Clutching functions upto htpy
when L = ε¹
S = X × S¹
In general, define "z" as this section
Similarly we can define:
for some
Bad picture
PLEASE
CHALKBOARD
WHEN YOUR CLASS
IS OVER

Bad picture

Similarly we can define:

$$z^{-1} \in \Gamma\pi^*(L^{-1})$$

$$z^k \in \Gamma\pi^*(L^{-k})$$

for some $a_k \in \Gamma(L^{-k})$

$$a_k z^k = \pi^* a_k \otimes z^k \in \pi^*(\mathcal{E}^1)$$

$$\begin{bmatrix} L^k = L^{\otimes k} \\ L^{-1} = L^* \end{bmatrix}$$

PLEASE ERASE THE CHALKBOARD WHEN YOUR CLASS IS OVER

Bad picture

Similarly, we can define:

$$z^{-1} \in \Gamma\pi^*(L^{-1})$$

$$z^k \in \Gamma\pi^*(L^{-k})$$

for some $a_k \in \Gamma(L^{-k})$

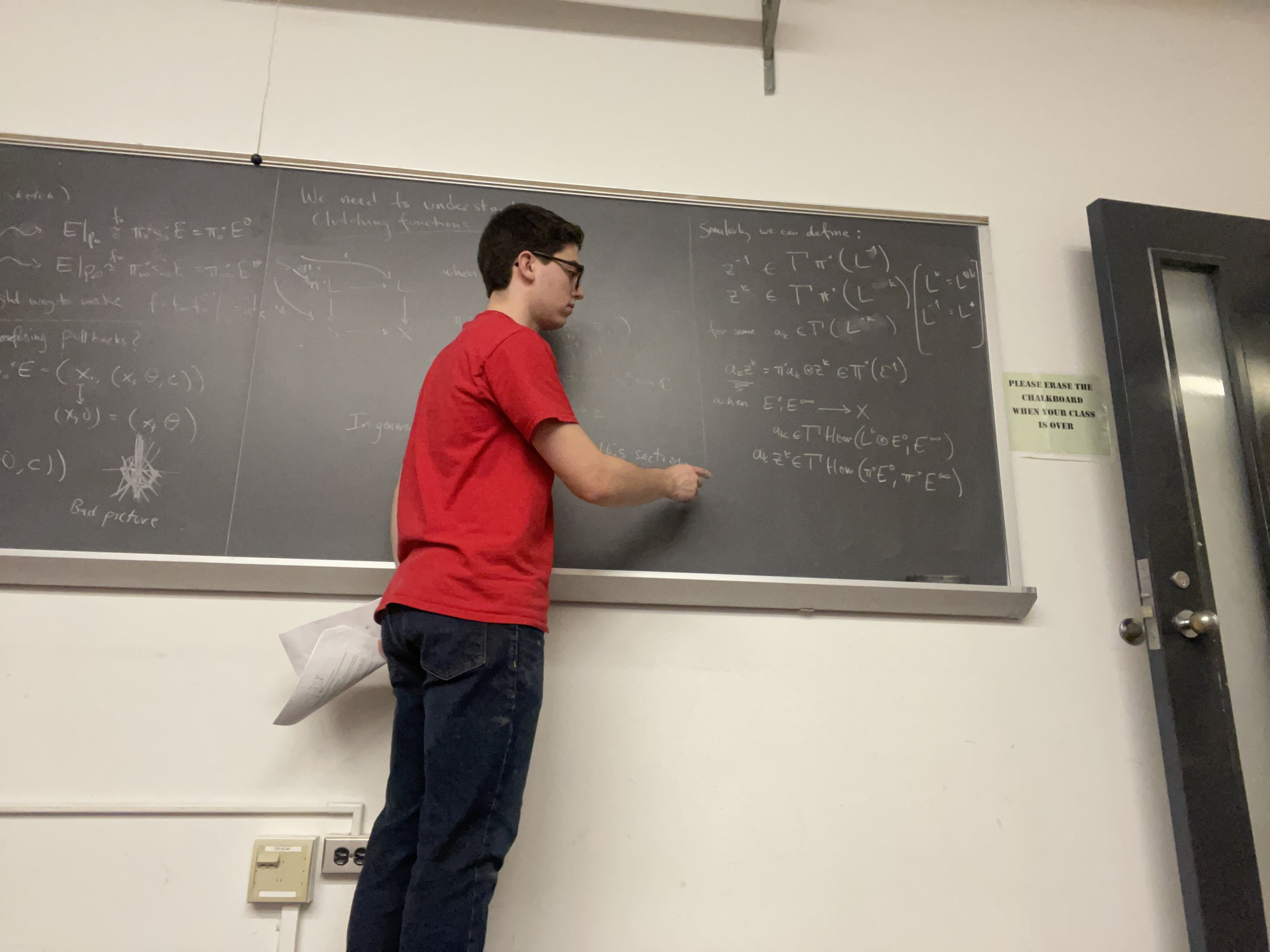
$$a_k z^k = \pi^* a_k \otimes z^k \in \Gamma\pi^*(\mathcal{E}^1)$$

when $E^0, E^\infty \longrightarrow X$

$$\begin{bmatrix} L^k = L^{\otimes k} \\ L^{-1} = L^* \end{bmatrix}$$

define "$z$" as this section

PLEASE ERASE THE CHALKBOARD WHEN YOUR CLASS IS OVER



Bad picture
Clutching functions
Similarly we can define:
for some
when
PLEASE ERASE THE CHALKBOARD WHEN YOUR CLASS IS OVER


We need to understand
Clutching functions upto htpy
when L = ε¹
S = X × S¹
Similarly we can define:
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as this section
Bad picture
PLEASE ERASE THE
CHALKBOARD
WHEN YOUR CLASS
IS OVER

Similarly we can define:

$$z^{-1} \in \Gamma \pi^* (L^{-1})$$

$$z^k \in \Gamma \pi^* (L^{-k}) \qquad \begin{bmatrix} L^k = L^{\otimes k} \\ L^{-1} = L^* \end{bmatrix}$$

for some $a_k \in \Gamma(L^{-k})$

$$a_k z^k = \pi^* a_k \otimes z^k \in \Pi^*(\mathcal{E}^1)$$

when $E^\circ, E^\infty \longrightarrow X$

$$a_k \in \Gamma \mathrm{Hom}(L^k \otimes E^\circ, E^\infty)$$

$$a_k z^k \in \Gamma \mathrm{Hom}(\pi^* E^\circ, \pi^* E^\infty)$$

$$f = \sum_{-n}^{n} a_k z^k \in \Gamma \mathrm{Hom}(\pi^* E^\circ, \pi^* E^\infty)$$

Def. a Laurent clutching func

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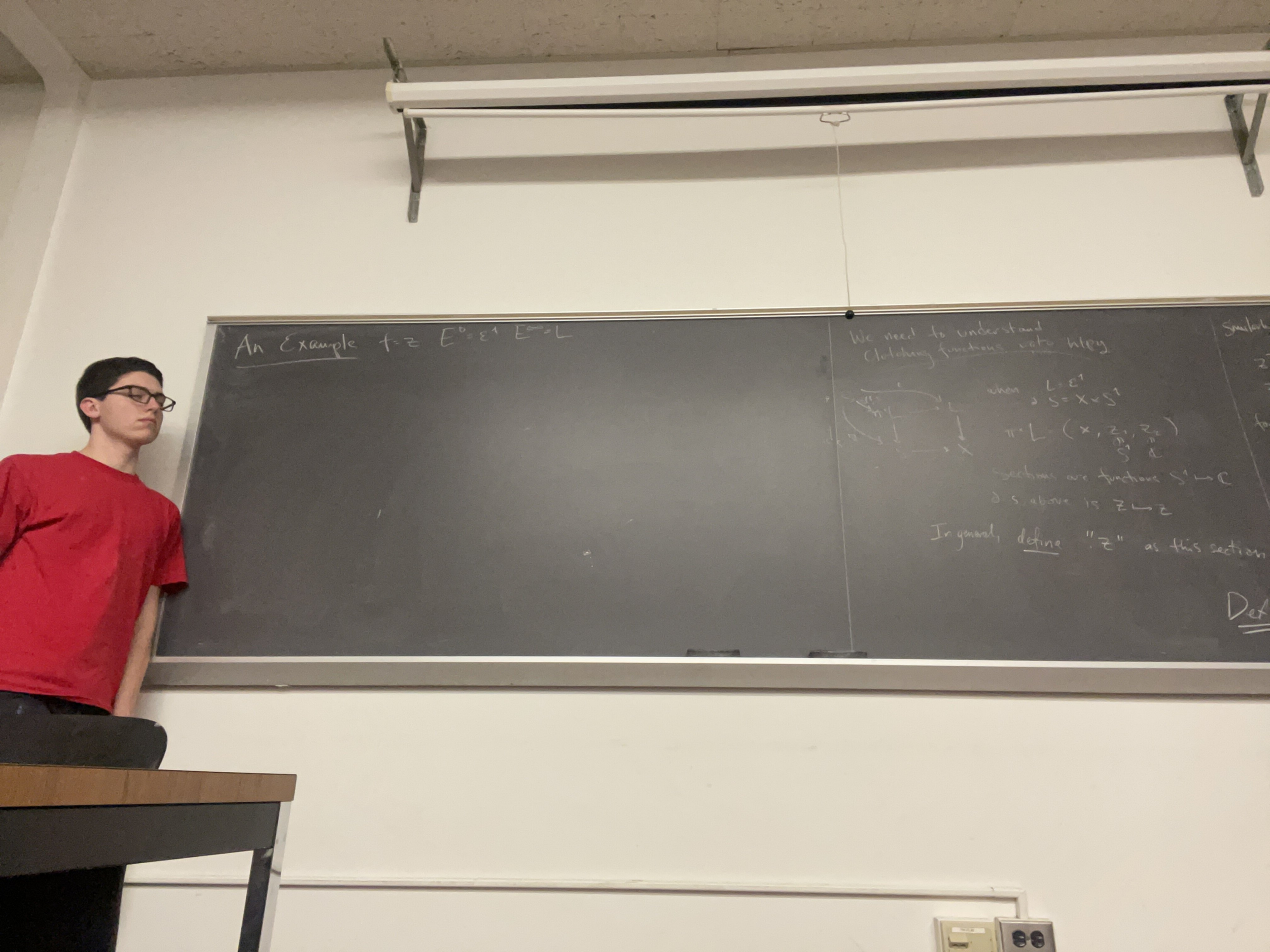
An Example


An Example f=z E^0=ε^1 E^∞=L
We need to understand
Clutching functions into htpy
when L=ε^1
∂S=X×S^1
π*L=(x, z_1, z_2)
sections are functions S^1 ↦ ℂ
∂S above is z ↦ z
In general, define "z" as this section
Def

An Example $f = z$ $E^0 = \varepsilon^1$ $E^\infty = L$

$H^*_\sigma(\varepsilon^1, z, L)$

We need to understand
(lifting functions into Hey

when $L = \varepsilon^1$
$\Rightarrow S = X \times S^1$

$\pi^* L \ (x, z_1, z_2)$

sections are functions $S^1 \to \mathbb{C}$

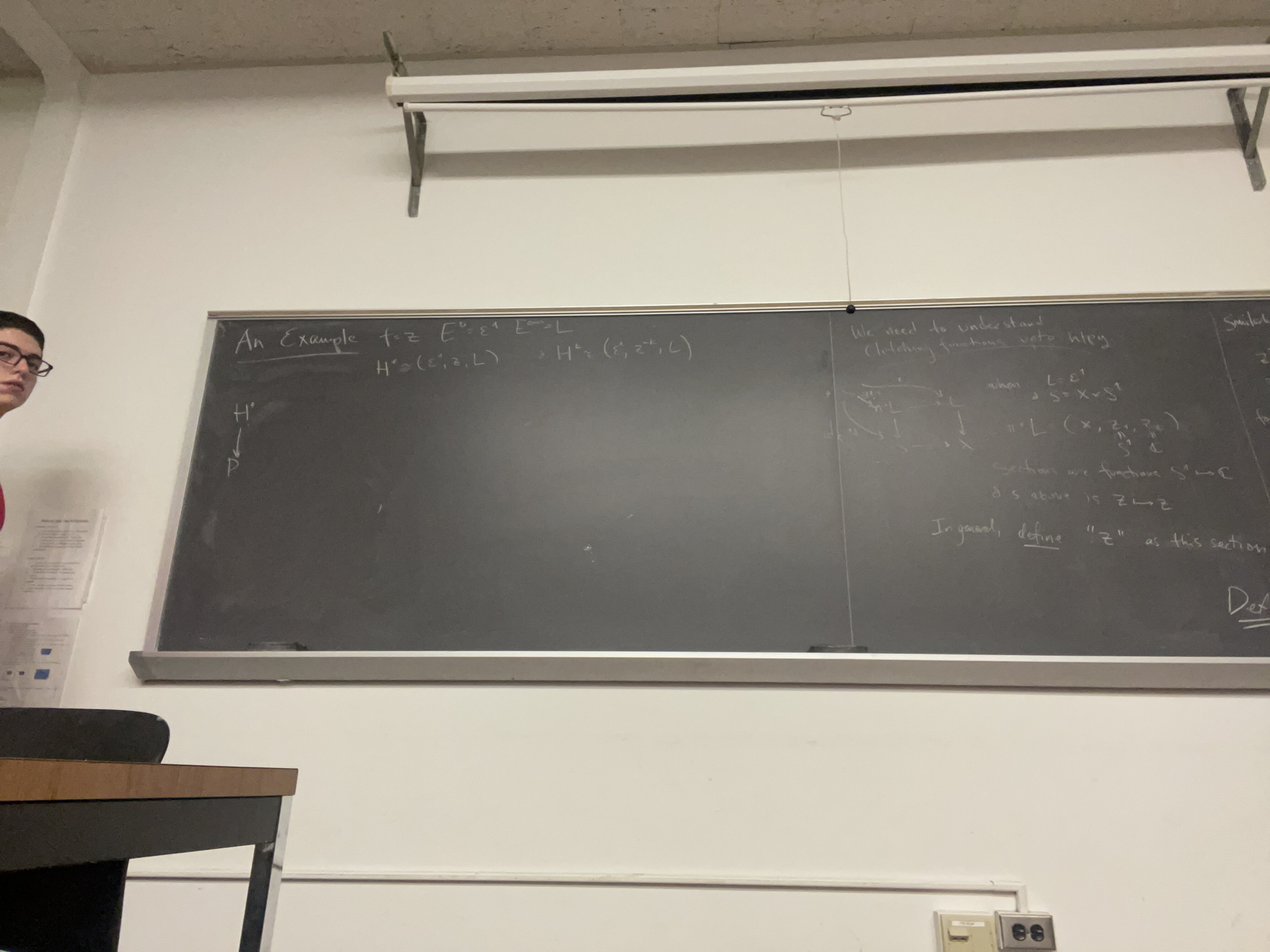
$\Rightarrow$ s above is $z \mapsto z$

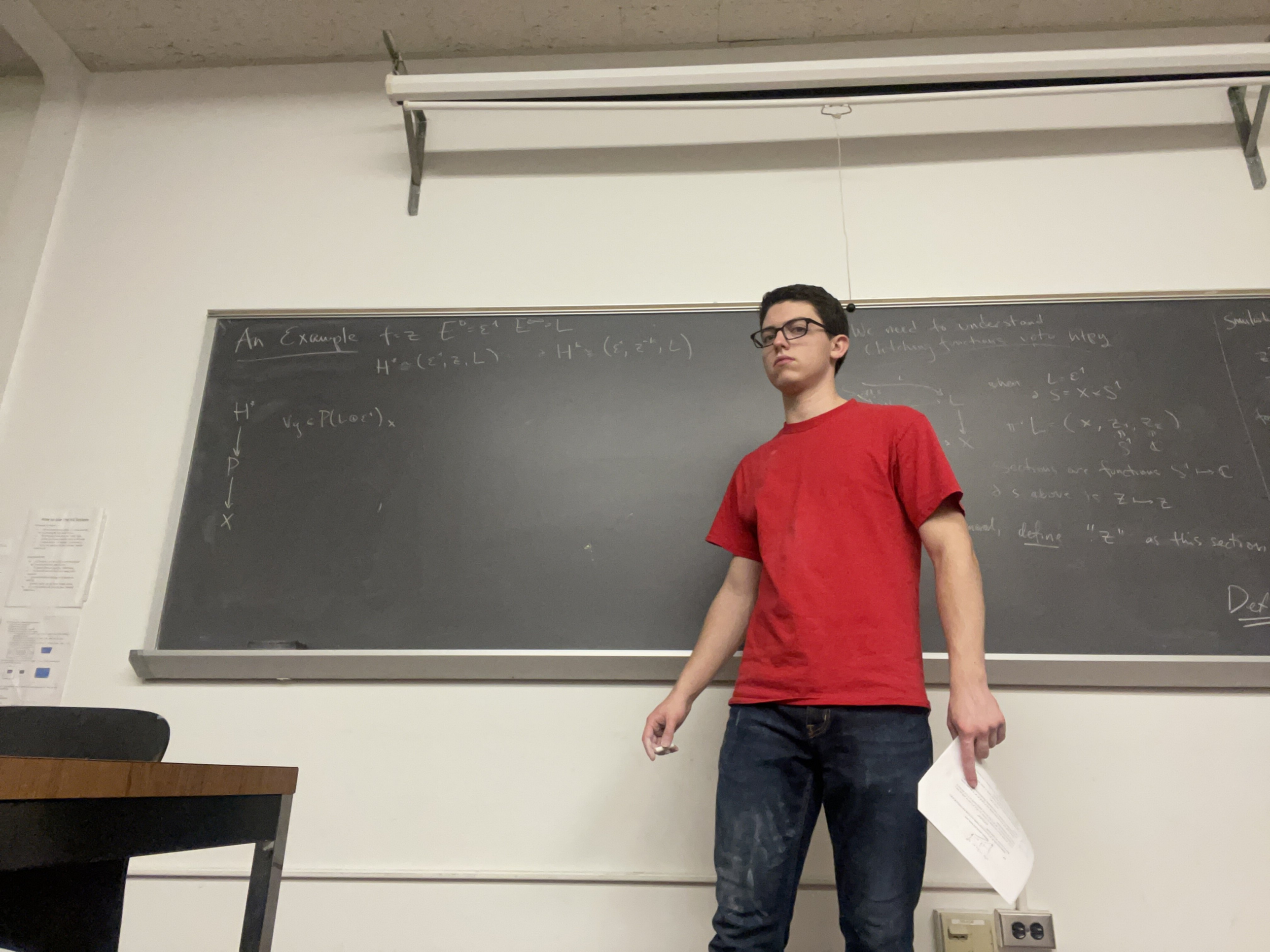
In general, define "z" as this section

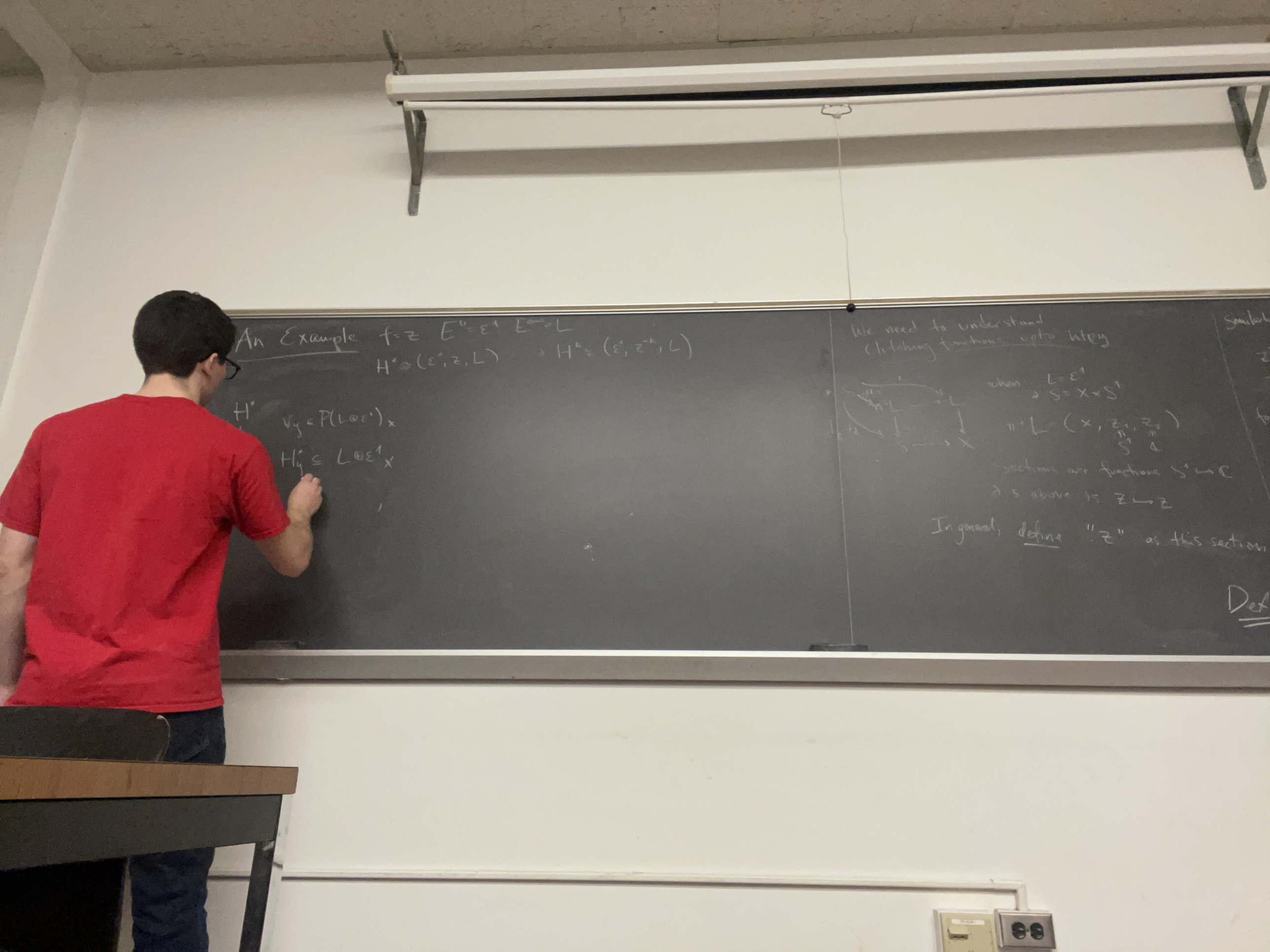
Def

An Example $t=z$ $E^0=\varepsilon^1$ $E^\infty=L$

$H^*=(\varepsilon^1, z, L)$ $H^*=(\varepsilon, z^{-1}, L)$


An Example f=z E^0=ε^1 E^∞=L
H^* ≅ (ε^1, z, L) ∂ H^k ≅ (ε^1, z^{-k}, L)
H^*
P
We need to understand
Clutching functions upto htpy
when L=ε^1
∂ S=X×S^1
π^*L (x, z_1, z_2)
S^1 ℂ
Sections are functions S^1 ↦ ℂ
∂ s above is z ↦ z
In general, define "z" as this section
Def


An Example
We need to understand
when
sections are functions
define "z" as this section


An Example f = z E^0 = ε^1 E^∞ = L
H^* ≅ (ε^1, z, L) ∂ H^k ≅ (ε^1, z^{-k}, L)
H^*
∀y ∈ P(L ⊗ ε^1)_x
H^*_y ⊆ L ⊗ ε^1_x
We need to understand
clutching functions upto htpy
when L = ε^1
∂ S = X × S^1
π^* L = (x, z_1, z_2)
S^1 ℂ
sections are functions S^1 ↦ ℂ
∂ s above is z ↦ z
In general, define "z" as this section
Def

An Example $f=z$ $E^0=\varepsilon^1$ $E^{-1}=L$

$H^1=(\varepsilon^1, z, L)$ $\rightsquigarrow$ $H^2=(\varepsilon^1, z^{-1}, L)$

$H'$

$\downarrow P$

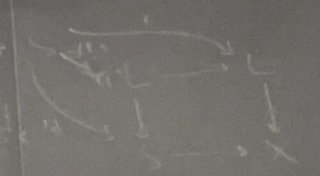
$X$

$V_y \subset P(L\oplus\varepsilon^1)_x$

$H'_y \subseteq L\oplus\varepsilon^1{}_x$

the line through $y$

We need to understand clutching functions very well



when $L=\varepsilon^1$

$\rightsquigarrow S=X\times S^1$

$\pi^*L = (x, z, z)$ ($S^1$, $\mathbb{C}$)

sections are functions $S^1\to\mathbb{C}$

$\rightsquigarrow$ s above is $z\mapsto z$

In general, define "$z$" as this section

Def

## An Example

$t=z$ $E^0=\varepsilon^1$ $E^\infty=L$

$H^1 \cong (\varepsilon^1, z, L)$ $\quad H^k \cong (\varepsilon^1, z^{-k}, L)$

$H^1$

$\downarrow$

$P$

$\downarrow$

$X$

$\forall y \in P(L \oplus \varepsilon^1)_x$

$H^1_y \subseteq L \oplus \varepsilon^1_x$

the line through $y$

$H^1_y = L_x \oplus 0 \Leftrightarrow y = \infty$

$H^1_y = 0 \oplus \varepsilon^1_x \Leftrightarrow y = 0$

## An Example $t=z$ $E^0=\varepsilon^1$ $E^\infty=L$

$H^* \cong (\varepsilon^1, z, L)$ $\quad H^k \cong (\varepsilon^1, z^{-k}, L)$

$H^*$

$\downarrow P$

$X$

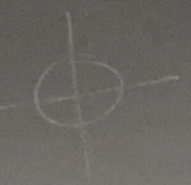
$\forall y \in P(L\oplus\varepsilon^1)_x$

$H^*_y \subseteq L\oplus\varepsilon^1 X$

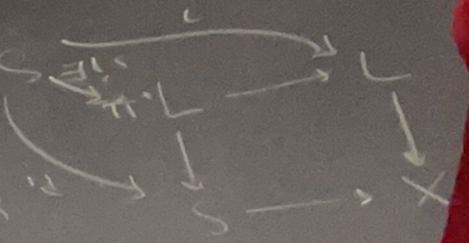
the line through $y$

$H^*_y = L_x \oplus 0 \Leftrightarrow y=\infty$

$H^*_y = 0\oplus\varepsilon^1_y \Leftrightarrow y=0$



We need to understand
Clutching functions




In gener

Example $t=z$ $E^0=\varepsilon^1$ $E^\infty=L$

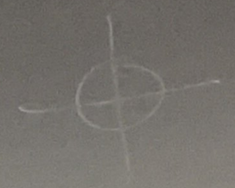
$H^* \cong (\varepsilon^1, z, L)$ $\quad H^k \cong (\varepsilon^1, z^{-k}, L)$

$\{y \in P(L\oplus\varepsilon^1)_x$

$\subseteq L\oplus\varepsilon^1_x$

$H^*_y = L_x \oplus 0 \Leftrightarrow y=\infty$

$H^*_y = 0\oplus\varepsilon^1_y \Leftrightarrow y=0$

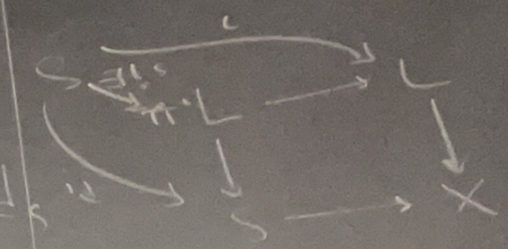


line through $y$

Thinking of $H^*$ as a subbundle of $\pi^*(L\oplus\varepsilon^1) \longrightarrow S$

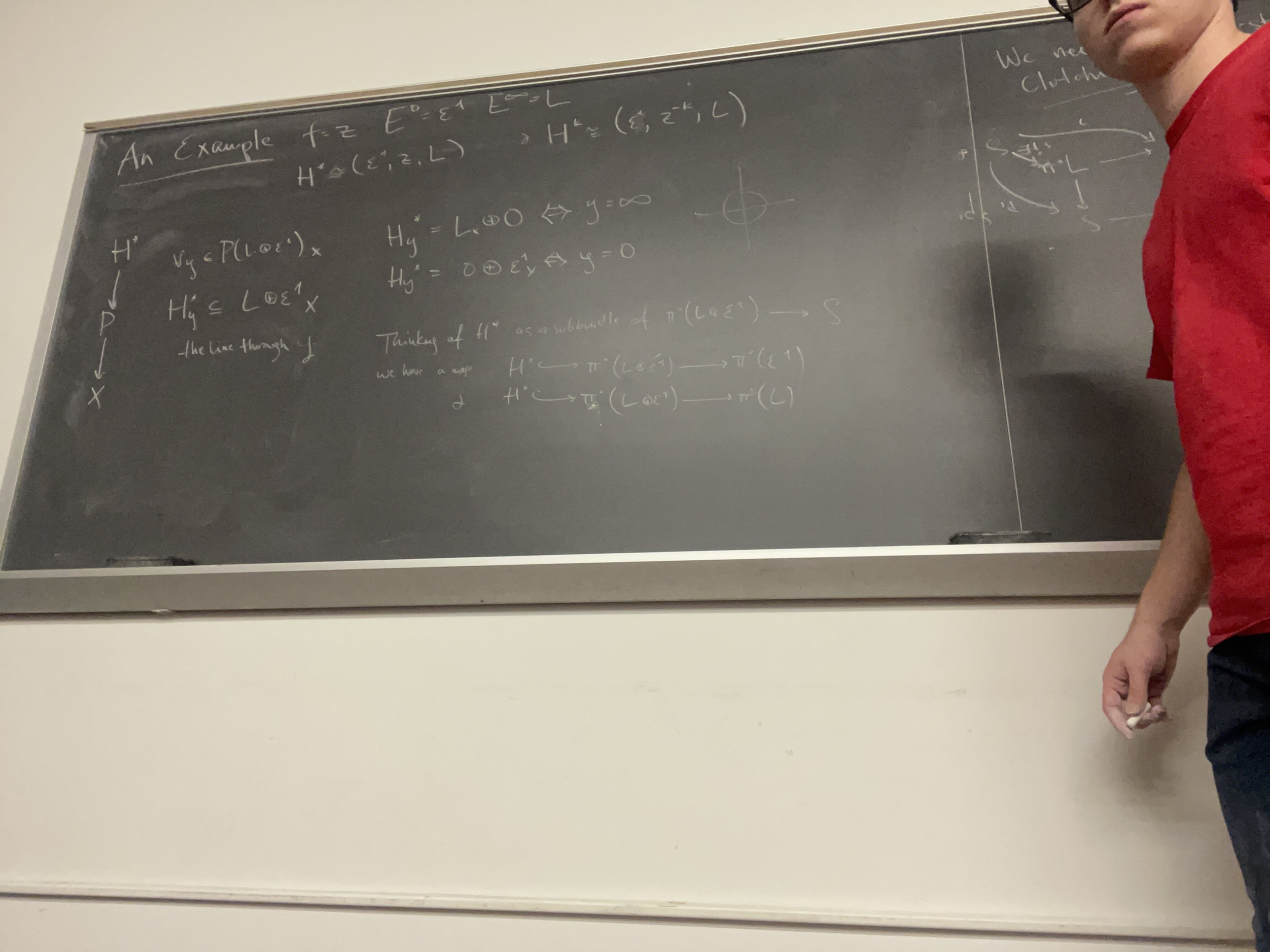
we have a map $H^* \hookrightarrow \pi^*(L\oplus\varepsilon^1) \longrightarrow \pi^*(\varepsilon^1)$

We need to understand Clutching functions




where

In general,


An Example
f = z
E^0 = ε^1
E^∞ = L
H^* ≅ (ε^1, z, L)
& H^k ≅ (ε^1, z^{-k}, L)
H^*
P
X
∀y ∈ P(L ⊕ ε^1)_x
H^*_y ⊆ L ⊕ ε^1_x
the line through y
H^*_y = L_x ⊕ 0 ⇔ y = ∞
H^*_y = 0 ⊕ ε^1_x ⇔ y = 0
Thinking of H^* as a subbundle of π^*(L ⊕ ε^1) ⟶ S
we have a map H^* ↪ π^*(L ⊕ ε^1) ⟶ π^*(ε^1)
& H^* ↪ π^*(L ⊕ ε^1) ⟶ π^*(L)
We nee
Clutch
S
π^*L
S
id_S

## An Example $f=z$ $E^0=\mathcal{E}^1$ $E^\infty=L$

$H^k \cong (\mathcal{E}^1, z^{-k}, L)$

$H^\vee \cong (\mathcal{E}^1, z, L)$

$H^\vee$ $\downarrow$ $P$ $\downarrow$ $X$

$V_y \subset P(L\oplus\mathcal{E}^1)_x$

$H^\vee_y \subseteq L\oplus\mathcal{E}^1{}_x$

the line through $y$

$H^\vee_y = L_x \oplus 0 \Leftrightarrow y=\infty$

$H^\vee_y = 0\oplus\mathcal{E}^1_y \Leftrightarrow y=0$

Thinking of $H^\vee$ as a subbundle of $\pi^*(L\oplus\mathcal{E}^1) \longrightarrow S$

we have a map $H^\vee \hookrightarrow \pi^*(L\oplus\mathcal{E}^1) \longrightarrow \pi^*(\mathcal{E}^1)$

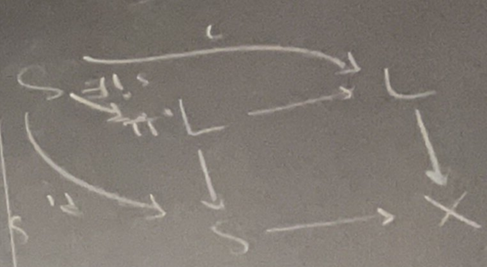
& $H^\vee \hookrightarrow \pi^*(L\oplus\mathcal{E}^1) \longrightarrow \pi^*(L)$

give

$f_0 : H^\vee|_{P_0} \to \pi^*(\mathcal{E}^1)$

$f_\infty : H^\vee|_{P_\infty} \to \pi^*(L)$

We need to understand Clutching functions




In general,

## An Example $f = z$

$E^0 = \mathcal{E}^1$ $E^\infty = L$

$H^\vee \cong (\mathcal{E}^1, z, L)$ $\rightsquigarrow H^k \cong (\mathcal{E}^1, z^{-k}, L)$

$H^\vee$

$\downarrow$

$P$

$\downarrow$

$X$

$\forall y \in P(L \oplus \mathcal{E}^1)_x$

$H^\vee_y \subseteq L \oplus \mathcal{E}^1_x$

the line through $y$

$H^\vee_y = L_x \oplus 0 \Leftrightarrow y = \infty$

$H^\vee_y = 0 \oplus \mathcal{E}^1_x \Leftrightarrow y = 0$

Thinking of $H^\vee$ as a subbundle of $\pi^*(L \oplus \mathcal{E}^1) \longrightarrow S$

we have a map $H^\vee \hookrightarrow \pi^*(L \oplus \mathcal{E}^1) \longrightarrow \pi^*(\mathcal{E}^1)$

& $H^\vee \hookrightarrow \pi^*(L \oplus \mathcal{E}^1) \longrightarrow \pi^*(L)$

which give isos

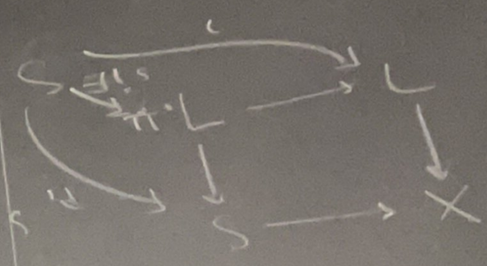
$f_0 : H^\vee|_{P_0} \to \pi^*(\mathcal{E}^1)$

$f_\infty : H^\vee|_{P_\infty} \to \pi^*(L)$

& a clutching func

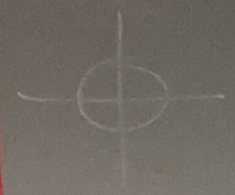
$f = f_\infty f_0^{-1}$ !

We need to understand
Clutching functions




In general,

$z^{-k}, L)$



$\varepsilon^1) \longrightarrow S$

$\longrightarrow \pi^{\sim}(\varepsilon^1)$

$\longrightarrow \pi^{\sim}(L)$

$\varepsilon^1)$

$L)$

a clutching func

$f = f_\infty \circ f_0^{-1}$ !

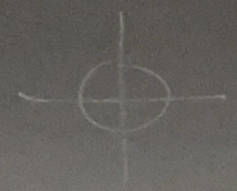
$f \; f_\infty f_0^{-1} \; \pi^{\sim}(\varepsilon^1) \longrightarrow \pi^{\sim}(L)$

$E^0 = \mathcal{E}^1$ $E^\infty = L$

$H^k = (\mathcal{E}^1, z^{-k}, L)$

$H_y^* = L_x \oplus 0 \Leftrightarrow y = \infty$

$H_y^* = 0 \oplus \mathcal{E}^1_y \Leftrightarrow y = 0$



Thinking of $H^*$ as a subbundle of $\pi^*(L \oplus \mathcal{E}^1) \longrightarrow S$

we have a map $H^* \hookrightarrow \pi^*(L \oplus \mathcal{E}^1) \longrightarrow \pi^*(\mathcal{E}^1)$

& $H^* \hookrightarrow \pi^*(L \oplus \mathcal{E}^1) \longrightarrow \pi^*(L)$

which give isos

$f_0 : H^*|_{P_0} \to \pi^*(\mathcal{E}^1)$

$f_\infty : H^*|_{P_\infty} \longrightarrow \pi^*(L)$

& a clutching func

$f = f_\infty f_0^{-1}$ !

$f \; f_\infty f_0^{-1} \; \pi^*(\mathcal{E}^1) \longrightarrow \pi^*(L)$

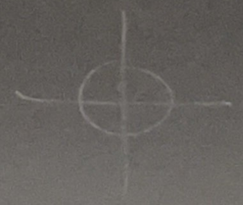
for $y \in S_x$, the graph of $f(y)$ is just $H_y^*$

Sy

$(\mathbb{C}, z, L) \rightsquigarrow H^k \cong (\mathbb{C}, z^{-k}, L)$

$H_y^* = L_x \oplus 0 \Leftrightarrow y = \infty$

$H_y^* = 0 \oplus \mathbb{C}^1_y \Leftrightarrow y = 0$



inking of $H^*$ as a subbundle of $\pi^*(L \oplus \mathbb{C}^1) \longrightarrow S$

ave a map $H^* \hookrightarrow \pi^*(L \oplus \mathbb{C}^1) \longrightarrow \pi^*(\mathbb{C}^1)$

$\rightsquigarrow H^* \hookrightarrow \pi_0^*(L \oplus \mathbb{C}^1) \longrightarrow \pi^*(L)$

give $f_0 : H^*|_{P_0} \to \pi^*(\mathbb{C}^1)$

$f_\infty : H^*|_{P_0} \to \pi^*(L)$

$\rightsquigarrow$ a clutching func

$f = f_\infty f_0^{-1}$ !

$f\ f_\infty f_0^{-1}\ \pi^*(\mathbb{C}^1) \to \pi^*(L)$

for $y \in S_x$ the graph of $f(y)$

is just $H_y^* = \langle y \otimes 1 \rangle \subseteq L_x \otimes 1_x$

Similarly we

$z^{-1}$

$z^k$

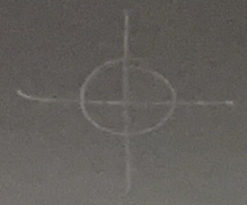
$a_k$

whe

$, z, L)$ & $H^k \cong (\varepsilon^1, z^{-k}, L)$

$H_y^* = L_x \oplus 0 \Leftrightarrow y = \infty$

$H_y^* = 0 \oplus \varepsilon^1_x \Leftrightarrow y = 0$



inking of $H^*$ as a subbundle of $\pi^*(L \oplus \varepsilon^1) \longrightarrow S$

have a map $H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(\varepsilon^1)$

& $H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(L)$

give $f_0 : H^*|_{P_0} \longrightarrow \pi^*(\varepsilon^1)$

$f_\infty : H^*|_{P_\infty} \longrightarrow \pi^*(L)$

& a clutching func $f = f_\infty f_0^{-1}$ !

$f \; f_\infty f_0^{-1} : \pi^*(\varepsilon^1) \longrightarrow \pi^*(L)$

for $y \in S_x$, the graph of $f(y)$ is just $H_y^* = \langle y \otimes 1 \rangle \subseteq L_x \otimes 1_x$

which means $f$ is the "$z$"-section

Similarly we

$z^{-1}$

$z^k$

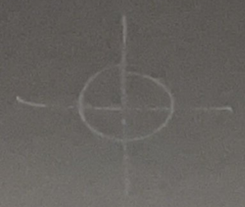
for some

$a_k$

whe

$\partial\ H^k \cong (\varepsilon^1, z^{-k}, L)$

$\oplus\ 0 \Leftrightarrow y = \infty$



$\oplus\ \varepsilon^1_x \Leftrightarrow y = 0$

$H^*$ as a subbundle of $\pi^*(L \oplus \varepsilon^1) \longrightarrow S$

$H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(\varepsilon^1)$

$H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(L)$

$f_0 : H^*|_{P_0} \longrightarrow \pi^*(\varepsilon^1)$

$f_\infty : H^*|_{P_\infty} \longrightarrow \pi^*(L)$

$\partial$ a clutching func

$f = f_\infty f_0^{-1}$ !

$f\ f_\infty f_0^{-1} : \pi^*(\varepsilon^1) \longrightarrow \pi^*(L)$

for $y \in S_x$, the graph of $f(y)$

is just $H_y^* = \langle y \otimes 1 \rangle \subseteq L_x \otimes 1_x$

which means $f$ is the "$z$"-section

$H^* = (\varepsilon^1, z, L)$

Similarly we

$z^{-1}$

$z^k$

for some

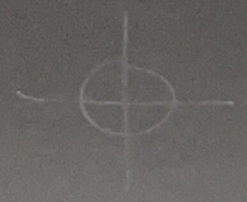
$\frac{a_k}{z}$

whe

$, z, L)$ & $H^k \cong (\varepsilon^1, z^{-k}, L)$

$H_y^* = L_x \oplus 0 \Leftrightarrow y = \infty$

$H_y^* = 0 \oplus \varepsilon^1_x \Leftrightarrow y = 0$



inking of $H^*$ as a subbundle of $\pi^*(L \oplus \varepsilon^1) \longrightarrow S$

have a map $H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(\varepsilon^1)$

& $H^* \hookrightarrow \pi^*(L \oplus \varepsilon^1) \longrightarrow \pi^*(L)$

give $f_0 : H^*|_{p_0} \longrightarrow \pi^*(\varepsilon^1)$

$f_\infty : H^*|_{p_\infty} \longrightarrow \pi^*(L)$

& a clutching func $f = f_\infty f_0^{-1}$ !

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for $y \in S_x$, the graph of $f(y)$ is just $H_y^* = \langle y \otimes 1 \rangle \subseteq L_x \otimes 1_x$

which means $f$ is the "$z$"-section

$H^* = (\varepsilon^1, z, L)$

remember my bad picture before?

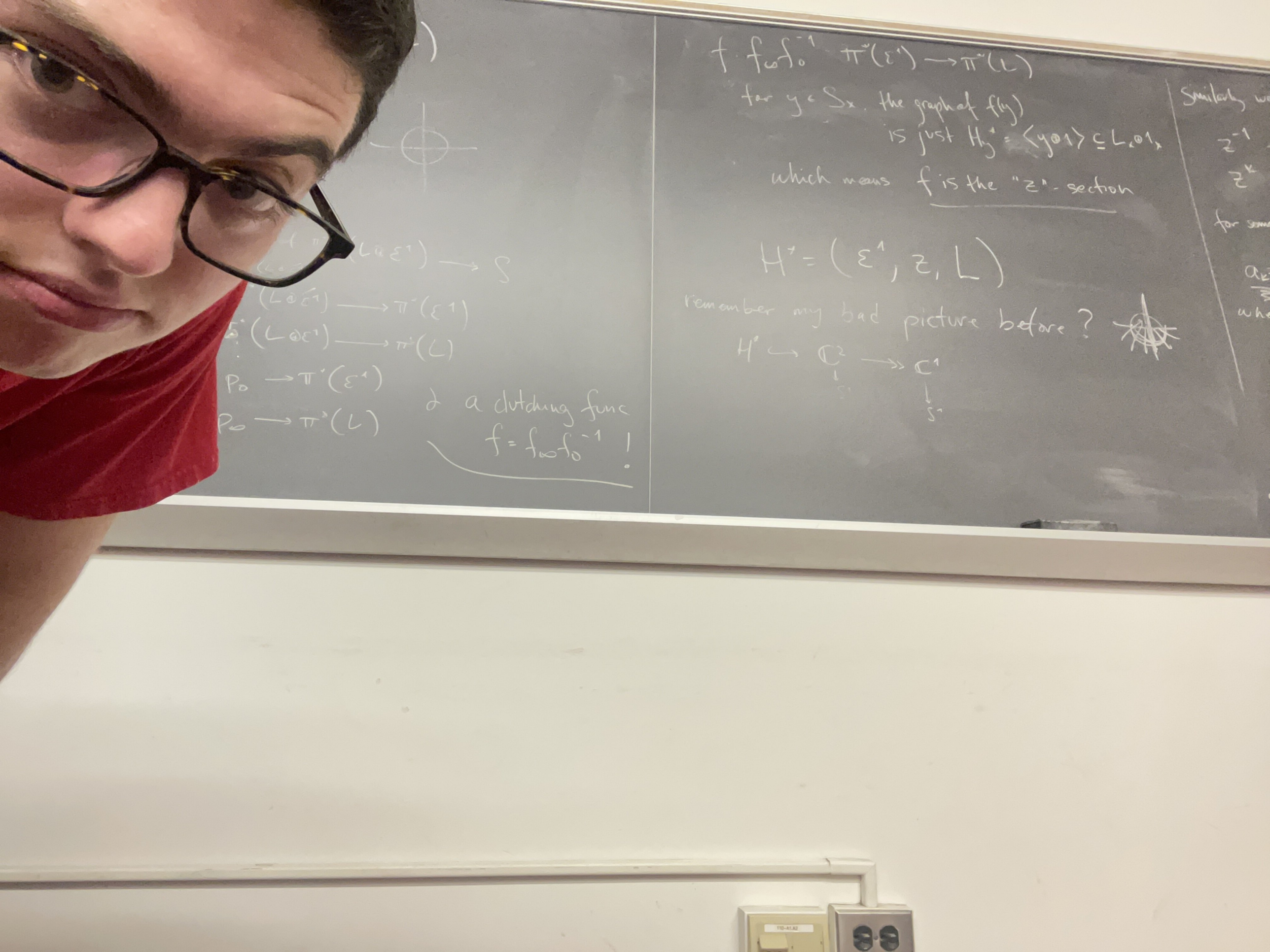


Similarly we

$z^{-1}$

$z^k$

wh



$(L \oplus \varepsilon^1) \longrightarrow S$
$(L \oplus \varepsilon^1) \longrightarrow \pi^*(\varepsilon^1)$
$(L \oplus \varepsilon^1) \longrightarrow \pi^*(L)$
$p_0 \longrightarrow \pi^*(\varepsilon^1)$
$p_\infty \longrightarrow \pi^*(L)$
& a clutching func
$f = f_\infty f_0^{-1}$ !
$f \cdot f_\infty f_0^{-1} \quad \pi^*(\varepsilon^1) \longrightarrow \pi^*(L)$
for $y \in S_x$, the graph of $f(y)$
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which means $f$ is the "$z$"-section
$H^* = (\varepsilon^1, z, L)$
remember my bad picture before?
$H^* \hookrightarrow \mathbb{C}^2 \twoheadrightarrow \mathbb{C}^1$
$S^1$
$S^1$
Similarly w
$z^{-1}$
$z^k$
for som
whe



Similarly we can define:

$$z^{-1} \in \Gamma \pi^*(L^{-1})$$

$$z^k \in \Gamma \pi^*(L^k) \quad \left[L^k := L^{\otimes k}\right]$$

$$\left[L^{-1} := L^*\right]$$

for some $a_k \in \Gamma(L^{-k})$

$$a_k z^k := \pi^* a_k \otimes z^k \in \pi^*(\mathcal{E}^1)$$

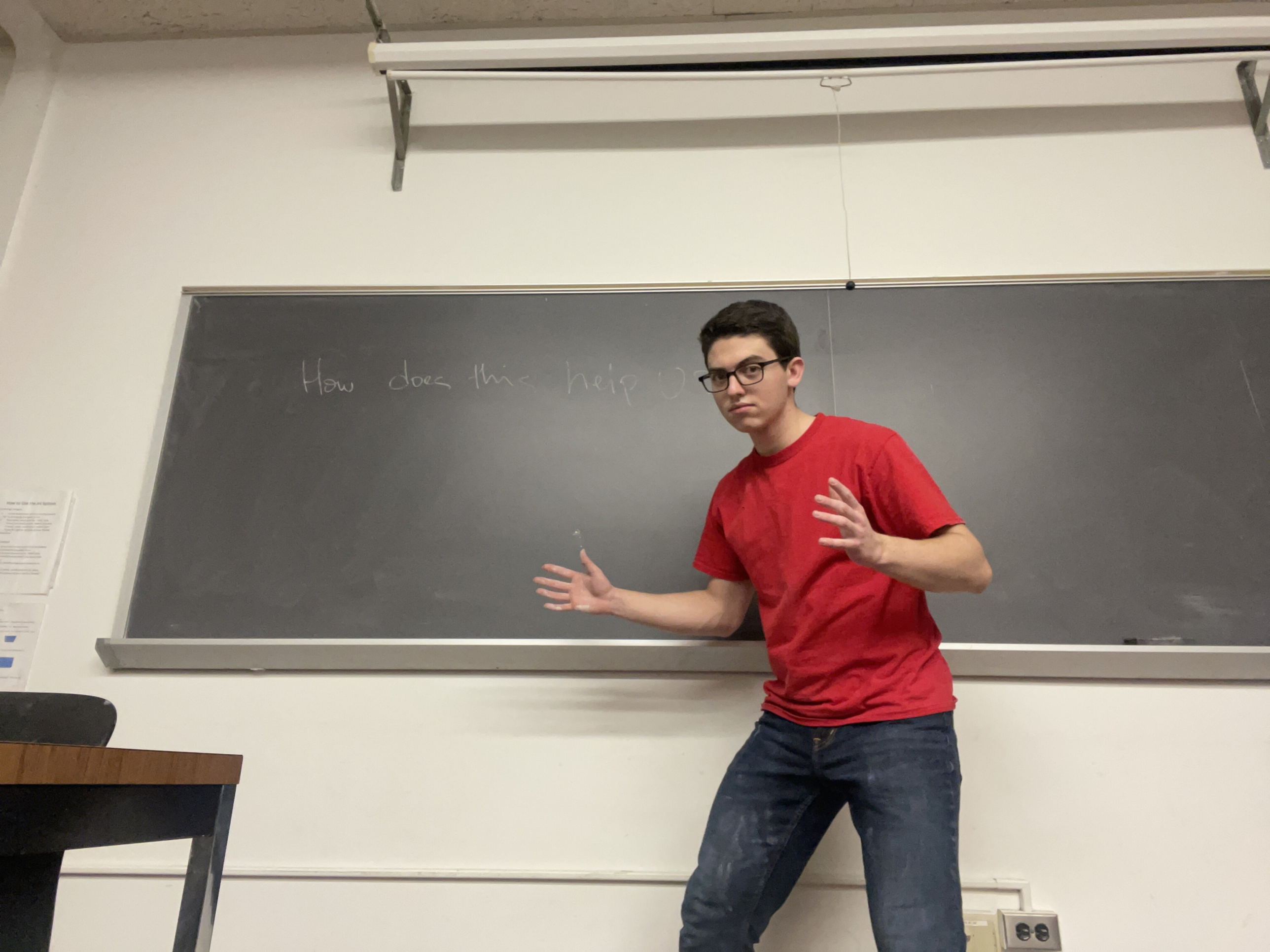
when $E^0, E^\infty \longrightarrow X$

$$a_k \in \Gamma \mathrm{Hom}(L^k \otimes E^0, E^\infty)$$

$$a_k z^k \in \Gamma \mathrm{Hom}(\pi^* E^0, \pi^* E^\infty)$$

$$f = \sum_{-n}^{n} a_k z^k \in \Gamma \mathrm{Hom}(\pi^* E^0, \pi^* E^\infty)$$

a Laurent clutching func.


How does this help

How does this help us?

How does this help us?

ANALYSIS

How does this help us?

ANALYSIS

given $f \in T'\mathrm{Hom}(L^k \otimes E^\circ; E^\infty)$

define Fourier coefficients.

given $f \in T'\mathrm{Hom}(L^k \odot E^\circ; E^\infty)$

define Fourier coefficients

$a_k \in T'\mathrm{Hom}(L^k \odot E^\circ; E^\infty)$

$$a_k(x) := \frac{1}{2\pi i} \int_{S_x} f_x z_x^{-k-1} \, dz_x$$

given $f \in T^1 Hom(L^k \odot E^\circ; E^\infty)$

define Fourier coefficients.

$a_k \in T^1 Hom(L^k \odot E^\circ; E^\infty)$

$a_k(x) := \frac{1}{2\pi i} \int_{S_x} f_x z_x^{-k-1} dz_x$

partial sums $S_n = \sum_{-n}^{n} a_k z^k$

given $f \in T'\mathrm{Hom}(L^k \otimes E^\circ; E^\infty)$

define Fourier coefficients

$a_k \in T'\mathrm{Hom}(L^k \otimes E^\circ; E^\infty)$

$$a_k(x) := \frac{1}{2\pi i} \int_{S_x} f_x z_x^{-k-1} \, dz_x$$

partial sums $\quad S_n = \sum_{-n}^{n} a_k z^k$

Cesaro means $\quad f_n = \frac{1}{n} \sum_{0}^{n} S_k$


given
By ANALYSIS

given $f \in T'$ Hom(

define Fourier coef

$a_k \in T'$

By Analysis

partial

Ces

given $f \in T'($ … $E^\infty)$

… $\otimes E^\circ, E^\infty)$

… $z^{-k-1} \, dz_x$

By Analysis

$f_n \longrightarrow f$ uniformly

$\Rightarrow$ is a Laurent clutching func for large enough $n$

given $f \in$ $E^\circ, E^\infty)$

efficients

$(L^k \otimes E^\circ, E^\infty)$

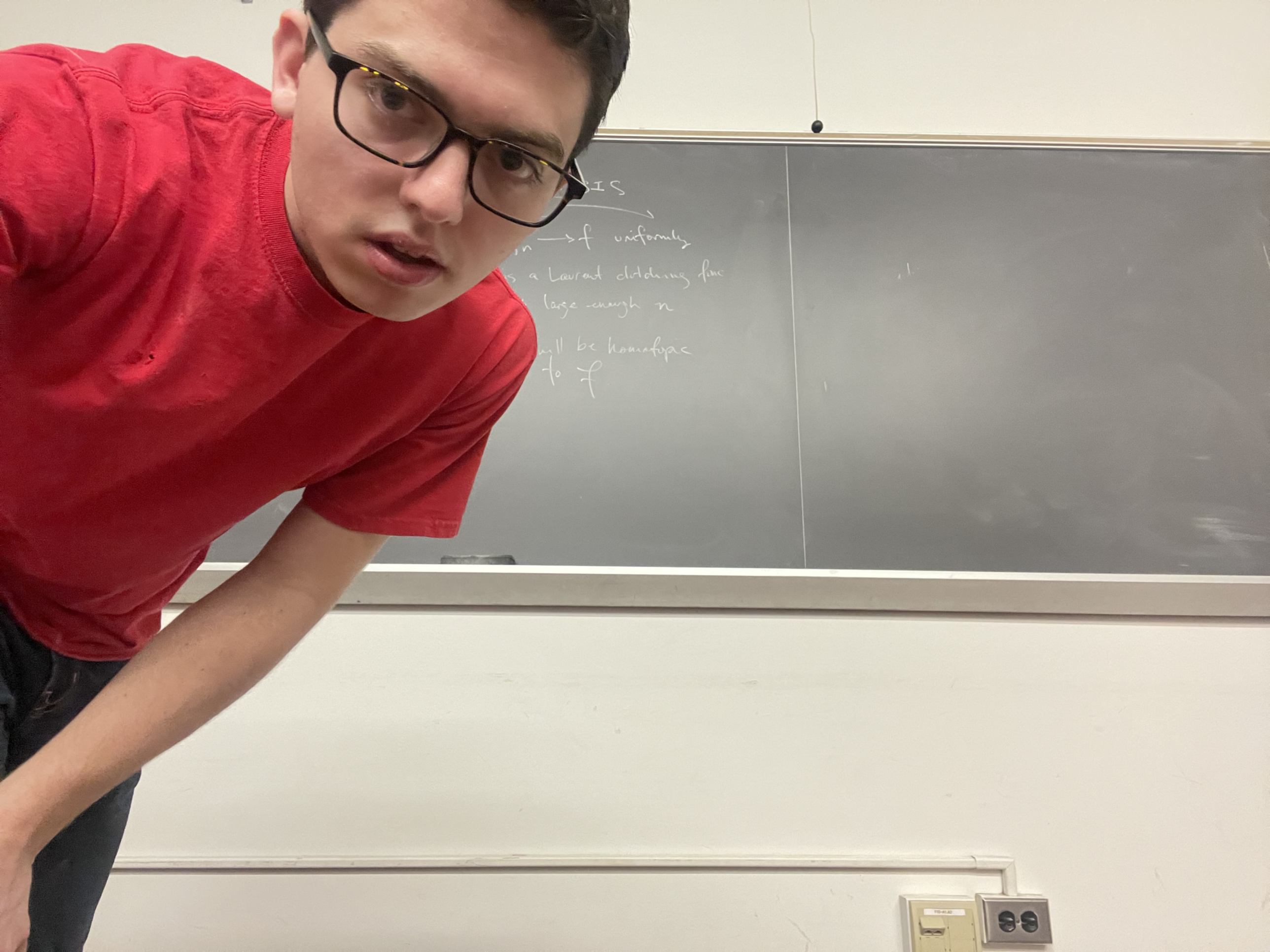
$f_x z_x^{-k-1} dz_x$

By Analysis

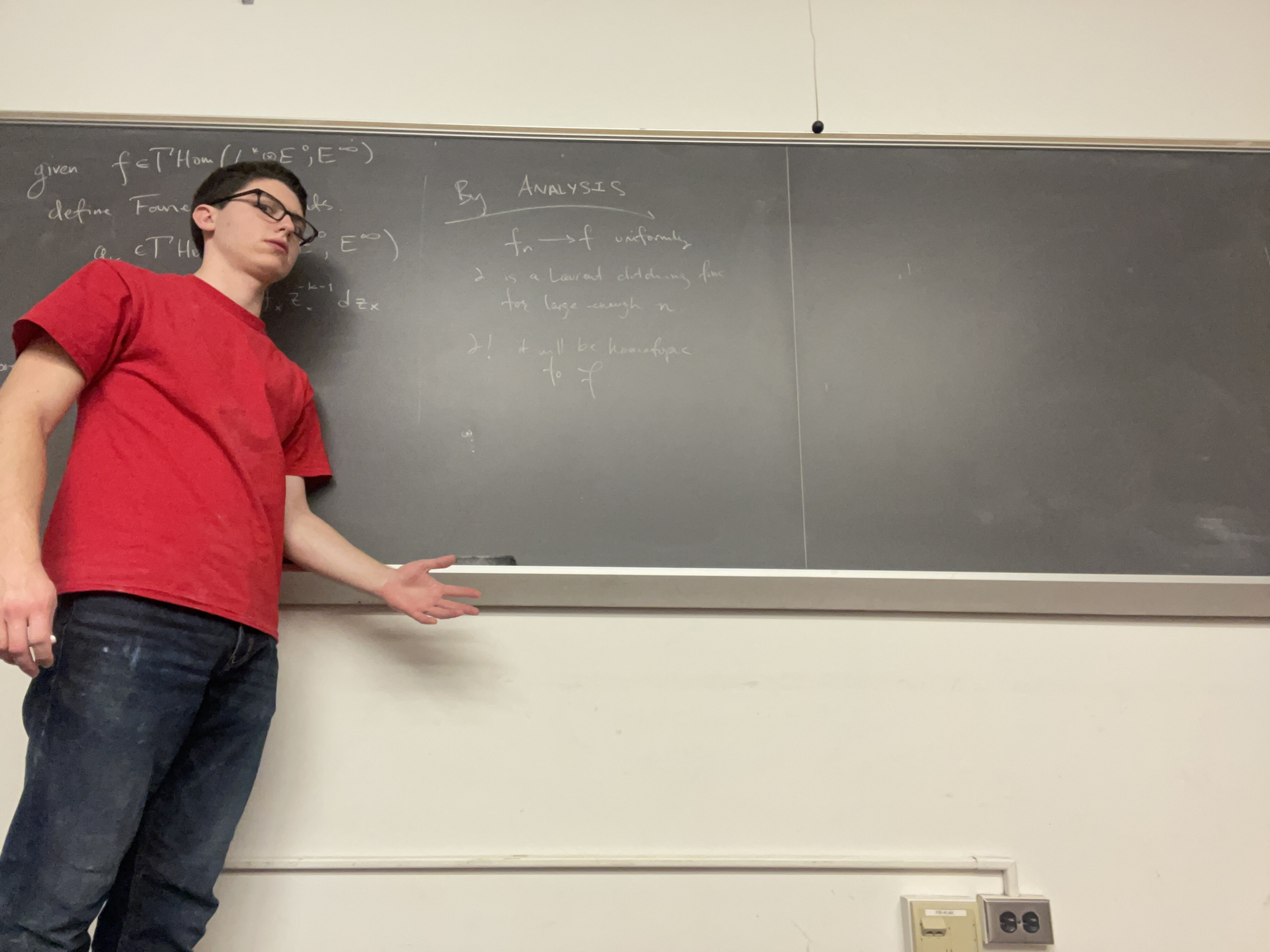
$f_n \to f$ uniformly

$f$ is a Laurent distributing func
for large enough $n$

I am not an analyst



→ f uniformly
a Laurent clutching func
large enough n
be homotopic
to f


given
By ANALYSIS
$f_n \to f$ uniformly
is a Laurent clutching func
for large enough $n$
it will be homotopic
to $f$

Now

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Now

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Now Suppose $P$ is linear

Prop $P$ is a linear clutching function for $E^0, E^\infty$
define endomorphisms $Q^0 \circlearrowright Q^\infty$ of $E^0 \circlearrowright E^\infty$

$$Q^0_x = \frac{1}{2\pi i}\int_{S_x} P_x^{-1} dP_x \circlearrowright Q^\infty = \frac{1}{2\pi i}\int_{S_\infty} dP_x^{-1} P_x$$

Now Suppose $\rho$ is linear

Prop $\rho$ is a linear clutching function for $E^0, E^\infty$

define endomorphisms $Q^0$ & $Q^\infty$ of $E^0$ & $E^\infty$

$$Q^0_x = \frac{1}{2\pi i}\int_{S_x} \rho_x^{-1} d\rho_x \quad \& \quad Q^\infty = \frac{1}{2\pi i}\int_{S_x} d\rho_x^{-1}\rho_x$$

Then $Q^0$ & $Q^\infty$ are projection operators (idempotent)

& $\rho Q^0 = Q^\infty \rho$, $E^2 = Q^2 E^1 \oplus (1-Q^2)E^2$

$= E^2_+ \oplus E^2_-$

&

$\rho = \rho_+ \oplus \rho_-$, $\rho_+$ an iso out of $S$, $\rho_-$ an iso inside $S$

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Now Suppose $P$ is linear

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define endomorphisms $Q^0, Q^\infty$ of $E^0, E^\infty$

$$Q^0_x = \frac{1}{2\pi i}\int_{S_x} P_x^{-1} dP_x \quad , \quad Q^\infty = \frac{1}{2\pi i}\int_{S_x} dP_x^{-1} P_x$$

Then $Q^0, Q^\infty$ are projection operators (idempotent)

& $PQ^0 = Q^\infty P$ ,

$$E^1 = Q^1 E^1 \oplus (1-Q^1)E^1$$
$$= E^1_+ \oplus E^1_-$$

$P = P_+ \oplus P_-$ , $P_+$ an iso out of $S$, $P_-$ an iso inside $S$

Corollary $p$ as above

Now Suppose $p$ is linea

Prop $p$ is a linea
for $E^0, E^\infty$
define endomo

$$Q^0_x = \frac{1}{2\pi i}\int$$

Then $Q$

& $pQ^0 =$

Corollary P as above

$P_+ = a_+ z + b_+$

$P_- = a_- z + b_-$

Corollary $P$ as above

$$P_+ = a_+ z + b_+$$
$$P_- = a_- z + b_-$$

$P^t : P_+^t + P_-^t$

$$P_+^t = a_+ z + t b_+$$
$$P_-^t = t a_+ z + b_-$$

Now Suppose $P$ is Linear

Prop $P$ is a Linear
for $E^0$, $E^\infty$
define endomo

$$Q^0_+ = \frac{1}{2\pi i}\int$$

Then

$$P Q^0 =$$

**Corollary** $p$ as above

$$p_+ = a_+ z + b_+$$
$$p_- = a_- z + b_-$$

$$p^t = p_+^t + p_-^t \quad \begin{cases} p_+^t = a_+ z + t b_+ \\ p_-^t = t a_+ z + b_- \end{cases}$$

gives a htpy of linear clutching functions $p \xrightarrow{\sim} a_+ z + b_-$

**Now** Suppose $p$ is linea

**Prop** $p$ is a linea
for $E^0, E^\infty$
define endo mo

$$Q^0_x = \frac{1}{2\pi i} \int$$

**Then** $Q$

& $pQ^0 =$

Corollary $p$ as above

$$p_+ = a_+ z + b_+$$
$$p_- = a_- z + b_-$$

$p^t : p_+^t + p_-^t$ ↙ ↘

$$p_+^t = a_+ z + t b_+$$
$$p_-^t = t a_+ z + b_-$$

gives a htpy of linear clutching functions $p \xrightarrow{\sim} a_+ z + b_-$

$$(E^\circ, p, E^\infty) \cong (E^\circ_+, z, L^{\otimes} E^\circ_+) \oplus (E^\circ_-, 1, E^\circ_-)$$

Suppose $p$ is linear

Corollary $p$ as above

$$p_+ = a_+ z + b_+$$
$$p_- = a_- z + b_-$$

$$p_+^t = a_+ z + t b_+$$
$$p_-^t = a_- z + b_-$$

$\to$ follows from prop

$p^t + p_-^t$

linear clutching

functions $p \xrightarrow{\sim} a_+ z + b_-$

$$(E^0, p, E^\infty) \cong (E^0_+, z, L \otimes E^0_+) \oplus (E^0_-, 1, E^0_-)$$

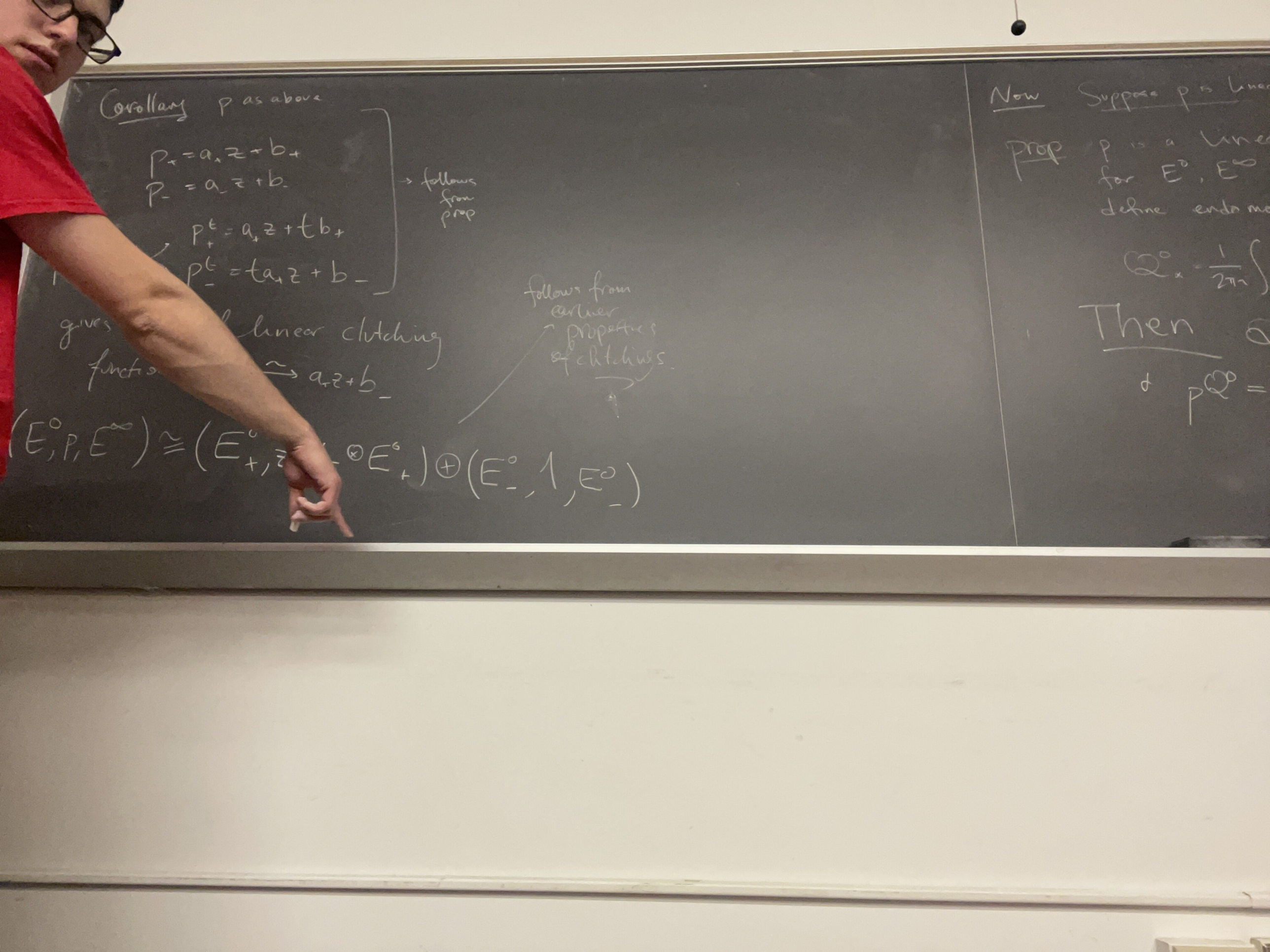
Now Suppose $p$ is linea

Prop $p$ is a linea
for $E^0, E^\infty$
define endomo

$$Q^0_x = \frac{1}{2\pi i} \int$$

Then $Q$

& $pQ^0 =$


Corollary p as above
$p_+ = a_+ z + b_+$
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$p_+^t = a_+ z + t b_+$
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→ follows from prop
gives
linear clutching
functio
$\xrightarrow{\sim} a_+ z + b_-$
follows from earlier properties of clutchings
$(E^0, p, E^\infty) \cong (E^0_+, z \otimes E^0_+) \oplus (E^0_-, 1, E^0_-)$
Now Suppose p is linea
Prop p is a line
for $E^0$, $E^\infty$
define endo mo
$Q^0_x = \frac{1}{2\pi i}\int$
Then
& $pQ^0 =$

**Corollary** $p$ as above

$$p_+ = a_+ z + b_+$$
$$p_- = a_- z + b_-$$

$$p^t = p_+^t + p_-^t \quad \begin{cases} p_+^t = a_+ z + t b_+ \\ p_-^t = t a_+ z + b_- \end{cases}$$

→ follows from prop

gives a htpy of linear clutching functions $p \xrightarrow{\sim} a_+ z + b_-$

follows from earlier properties of clutchings

$$(E^0, p, E^\infty) \cong (E^0_+, z, L \otimes E^0_+) \oplus (E^0_-, 1, E^0_-)$$
$$(E^0_+, a_+ z, E^\infty_+) \oplus (E^0_-, b_-, E^\infty_-)$$

**Now** Suppose $p$ is linea

**Prop** $p$ is a linea
for $E^0, E^\infty$
define endo mo

$$Q^0_x = \frac{1}{2\pi i} \int$$

**Then** $Q$

& $pQ^0 =$

follows from prop

A secret up my sleave...

A secret up my sleave..

There is a linearization procedure.

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A secret up my sleave..

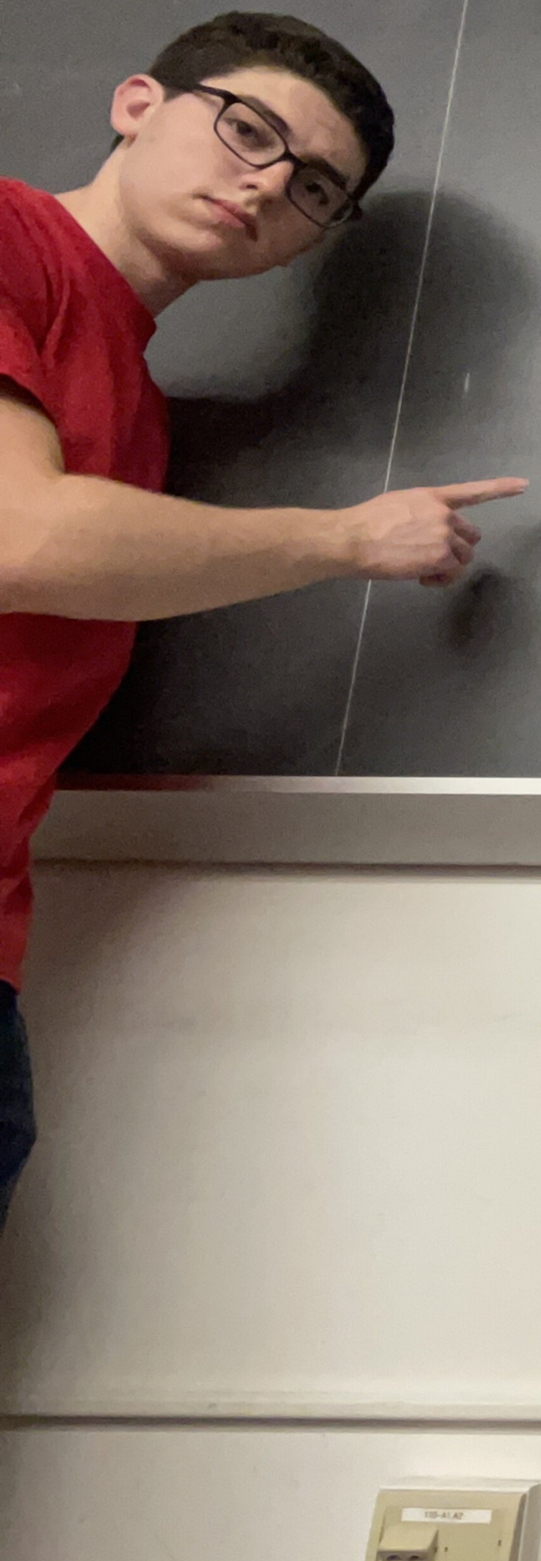
There is a linearization procedure:

$$P = \sum_{k=0}^{n} a_k z^k \quad \text{for } (E^0, E^\infty)$$

$$\Downarrow$$

$$\mathcal{L}^n(P) \cdot \pi'\left(\Sigma_{k=0}^{n} L^k \otimes E^0\right) \to \pi'\left(E^\infty \oplus \Sigma_{k=1}^{n} L^k \otimes E^0\right)$$

Linear in $z$



A secret up my sleave..

There is a linearization procedure:

$$P = \sum_{k=0}^{n} a_k z^k \text{ for } (E^0, E^\infty)$$

$$\Big\downarrow$$

$$\mathcal{L}^n(P) : \pi^*\left(\sum_{k=0}^{n} L^k \otimes E^0\right) \to \pi^*\left(E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right)$$

Linear in $z$

Prop

$$\left(E^0, P, E^\infty\right) \oplus \left(\sum_{k=1}^{n} L^k \otimes E^0, 1, \hat{\sum}_{k=0} L^k \otimes E^0\right)$$

$$\cong \left(\sum_{k=0}^{n} L^k \otimes E^0,\ \mathcal{L}^n(P),\ E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right)$$

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A secret up my sleave..

There is a linearization procedure:

$$P : \sum_{k=0}^{n} a_k z^k \text{ for } (E^\circ, E^\infty)$$

$$\Downarrow$$

$$\mathcal{L}^n(P) \quad \pi^*\left(\Sigma_{k=0}^{n} L^k \otimes E^\circ\right) \to \pi^*\left(E^\infty \otimes \sum_{k=1}^{n} L^k \otimes E^\circ\right)$$

linear in $z$

Prop

$$\left(E^\circ, P, E^\infty\right) \oplus \left(\hat{\sum_{k=1}^{n}} L^k \otimes E^\circ, 1, \hat{\sum_{k=2}} L^k \otimes E^\circ\right)$$

$$\cong \left(\sum_{k=0}^{n} L^k \otimes E^\circ, \mathcal{L}^n(P), E^\infty \otimes \hat{\sum_{k=1}^{n}} L^k \otimes E^\circ\right) =: \mathcal{L}^n\left(E^\circ, P, E^\infty\right)$$

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A secret up my sleave..

There is a linearization procedure:

$$P = \sum_{k=0}^{n} a_k z^k \text{ for } (E^0, E^\infty)$$

$$\Downarrow$$

$$\mathcal{L}^n(P)\ \pi'\left(\Sigma_{k=0}^{n} L^k \otimes E^0\right) \to \pi'\left(E^\infty \oplus \Sigma_{k=1}^{n} L^k \otimes E^0\right)$$

Linear in $z$

Separating out all the powers

Prop

$$\left(E^0, P, E^\infty\right) \oplus \left(\hat{\sum_{k=1}^{n}} L^k \otimes E^0,\ 1,\ \hat{\sum_{k\geq 2}} L^k \otimes E^0\right)$$

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A secret up my sleeve..

There is a linearization procedure:

$$P = \sum_{k=0}^{n} a_k z^k \text{ for } (E^0, E^\infty)$$

$$\Downarrow$$

$$\mathcal{L}^n(P)\ \pi^*\left(\Sigma_{k=0}^{n} L^k \otimes E^0\right) \to \pi^*\left(E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right)$$

Linear in $z$

Separating out all the powers like an order $n$ ODE

Prop

$$\left(E^0, P, E^\infty\right) \oplus \left(\sum_{k=1}^{n} L^k \otimes E^0, 1, \hat{\sum_{k\geq 2}} L^k \otimes E^0\right)$$

$$\cong \left(\sum_{k=0}^{n} L^k \otimes E^0,\ \mathcal{L}^n(P),\ E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right) =: \mathcal{L}^n\left(E^0, P, E^\infty\right)$$

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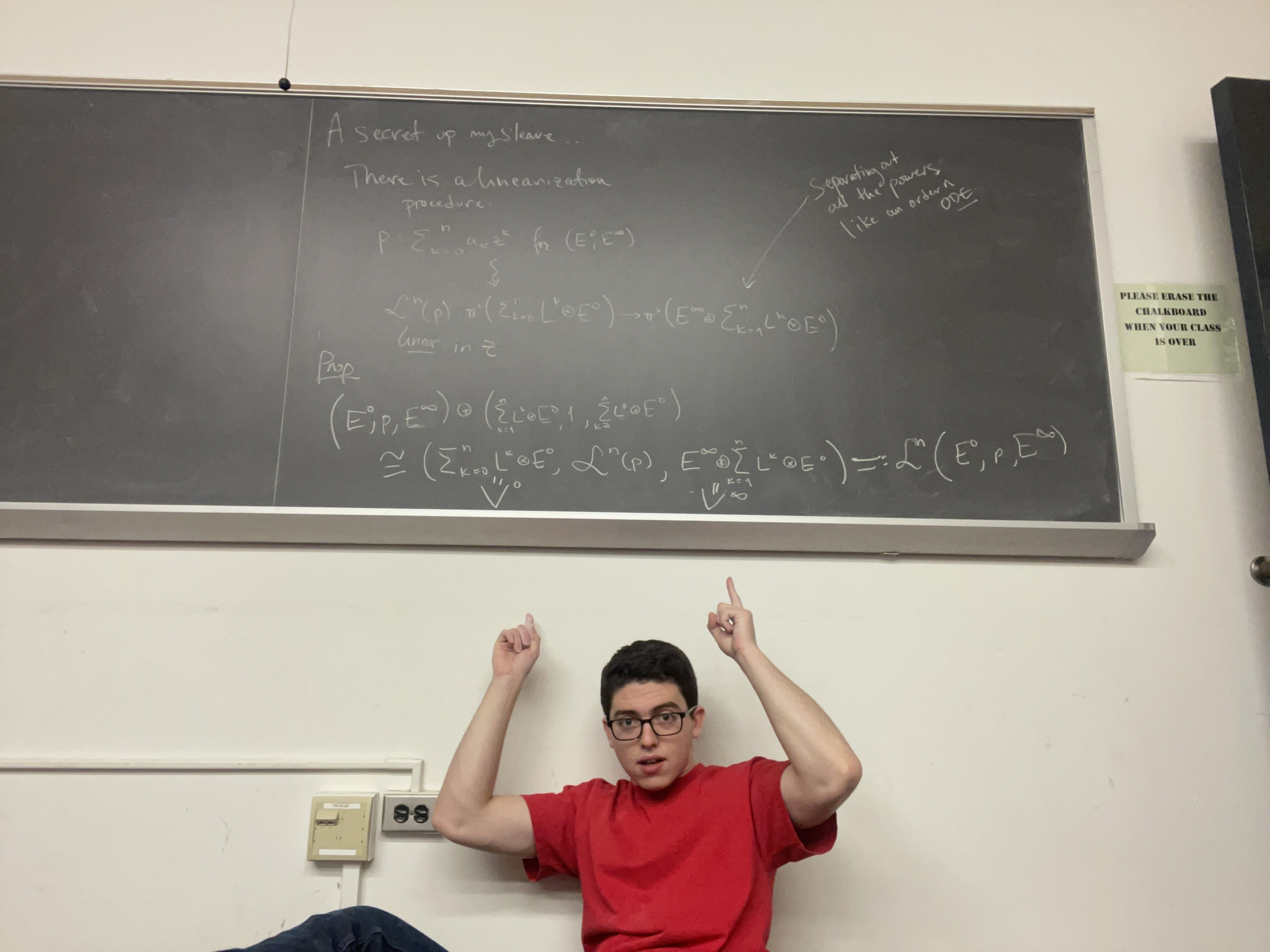
A secret up my sleave ..
There is a linearization procedure:
$p = \sum_{k=0}^{n} a_k z^k$ for $(E^0, E^\infty)$
$\mathcal{L}^n(p) : \pi^*\left(\sum_{k=0}^{n} L^k \otimes E^0\right) \to \pi^*\left(E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right)$
linear in $z$
Separating out all the powers like an order n ODE
Prop
$(E^0, p, E^\infty) \oplus \left(\sum_{k=1}^{n} L^k \otimes E^0, 1, \sum_{k=0}^{n} L^k \otimes E^0\right)$
$\cong \left(\sum_{k=0}^{n} L^k \otimes E^0, \mathcal{L}^n(p), E^\infty \oplus \sum_{k=1}^{n} L^k \otimes E^0\right) =: \mathcal{L}^n(E^0, p, E^\infty)$
$V^0$
$V^\infty$
PLEASE ERASE THE CHALKBOARD WHEN YOUR CLASS IS OVER

Using our projections

$V^\circ = V_+^\circ \oplus V_-^\circ$

A secret up my sleeve...

There is a linearization procedure:

$p = \sum_{k=0}^{n} a_k z^k$ for $(E^\circ, E$

$\mathcal{L}^n(p) \cdot \pi'\left(\sum_{k=0}^{n} L^k \otimes E^\circ\right.$

Linear in $z$

Prop

$\left(E^\circ, p, E^\infty\right) \oplus \left(\sum_{k=1} L^k \otimes E^\circ, 1\right.$

$\cong \left(\sum_{k=0}^{n} L^k \otimes E^\circ,\right.$

$V^\circ$

Using our projections.

$V^\circ = V_+^\circ \oplus V_-^\circ$

for clarity: $V_+^\circ = V_n(E^\circ, P, E^\infty)$

↑ comes from $\mathcal{L}^n(P)$

A secret up my sleave...

There is a linearization procedure:

$P = \sum_{k=0}^{n} a_k z^k$ for $(E^\circ, E$

$\mathcal{L}^n(P) \cdot \pi'\left(\sum_{k=0}^{n} L^k \otimes E^\circ\right.$

Linear in $z$

Prop

$\left(E^\circ, P, E^\infty\right) \oplus \left(\sum_{k=1}^{n} L^k \otimes E^\circ, 1\right.$

$\cong \left(\sum_{k=0}^{n} L^k \otimes E^\circ, \right.$

$V^\circ$

Using our projections

$$V^0 = V^0_+ \oplus V^0_-$$

for clarity: $V^0_+ = V_n(E^0, p, E^\infty)$

comes from $\mathcal{L}^n(p)$

Some properties of the linearization

A secret up my sleave...

There is a linearization procedure:

$p = \sum_{k=0}^{n} a_k z^k$ for $(E^0, E$

$\rightsquigarrow$

$\mathcal{L}^n(p) = \pi^*(\sum_{k=0}^{n} L^k \otimes E^0$

linear in $z$

Prop

$$(E^0, p, E^\infty) \oplus \left(\sum_{k=1}^{n} L^k \otimes E^0, 1\right.$$

$$\cong \left(\sum_{k=0}^{n} L^k \otimes E^0, \right.$$

$$\overset{\shortparallel}{V^0}$$

Using our projections

$V^\circ = V_+^\circ \oplus V_-^\circ$

for clarity: $V_+^\circ = V_n(E^\circ, P, E^\infty)$

↑ comes from $\mathcal{L}^\circ(P)$

Some properties of the linearization

1) $P_0 \simeq P_1$ poly clutch fibers $\Rightarrow V_n(E^\circ, P_0, E^\infty) \cong V_n(E^\circ, P_1, E^\infty)$

A secret up my sleave...

There is a linearization procedure:

$P: \sum_{k=0}^{n} a_k z^k$ for $(E^\circ,$

$\Downarrow$

$\mathcal{L}^n(P)$ $\pi^*(\sum_{k=0}^{n} L^k \otimes E^\circ$

Linear in $z$

Prop

$(E^\circ, P, E^\infty) \oplus (\sum_{k=1}^{n} L^k \otimes E^\circ,$

$\cong (\sum_{k=0}^{n} L^k \otimes E^\circ,$

$= V^\circ$

Using our projections

$V^\circ = V_+^\circ \oplus V_-^\circ$

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↑ comes from $\mathcal{L}^n(P)$

Some properties of the linearization

1) $P_0 \simeq P_1$ path clutch fixes $\Rightarrow V_n(E^\circ, P_0, E^\infty) \cong V_n(E^\circ, P_1, E^\infty)$

2) $V_{n+1}(E^\circ, P, E^\infty) \cong V_n(E^\circ, P, E^\infty)$

A secret up my sleeve...

There is a linearization procedure:

$P = \sum_{k=0}^{n} a_k z^k$ for $(E^\circ$

$\Leftrightarrow$

$\mathcal{L}^n(P)$ $\pi'(\Sigma_{k=0}^{n} L^k \otimes E$

Linear in $z$

Prop

$(E^\circ, P, E^\infty) \oplus (\sum_{k=1}^{n} L^k \otimes E^\circ,$

$\cong (\sum_{k=0}^{n} L^k \otimes E^\circ)$

$= V^\circ$

Using our projections

$V^\circ = V_+^\circ \oplus V_-^\circ$

for clarity: $V_+^\circ = V_n(E^\circ, P, E^\infty)$

↑ comes from $\mathcal{L}^n(P)$

Some properties of the linearization

1) $P_0 \simeq P_1$ path clutch fixes $\Rightarrow V_n(E^\circ, P_0, E^\infty) \cong V_n(E^\circ, P_1, E^\infty)$

2) $V_{n+1}(E^\circ, P, E^\infty) \cong V_n(E^\circ, P, E^\infty)$

3) $V_{n+1}(L^{-1} \otimes E^\circ, zP, E^\infty) \cong V_n(E^\circ, P, E^\infty) \oplus (L^{-1} \otimes E^\circ)$

A secret up my sleave...

There is a linearization procedure:

$P = \sum_{k=0}^{n} a_k z^k$ for $(E^\circ,$

$\downarrow$

$\mathcal{L}^n(P)$ $\pi'(\Sigma_{k=0}^{n} L^k \otimes E$

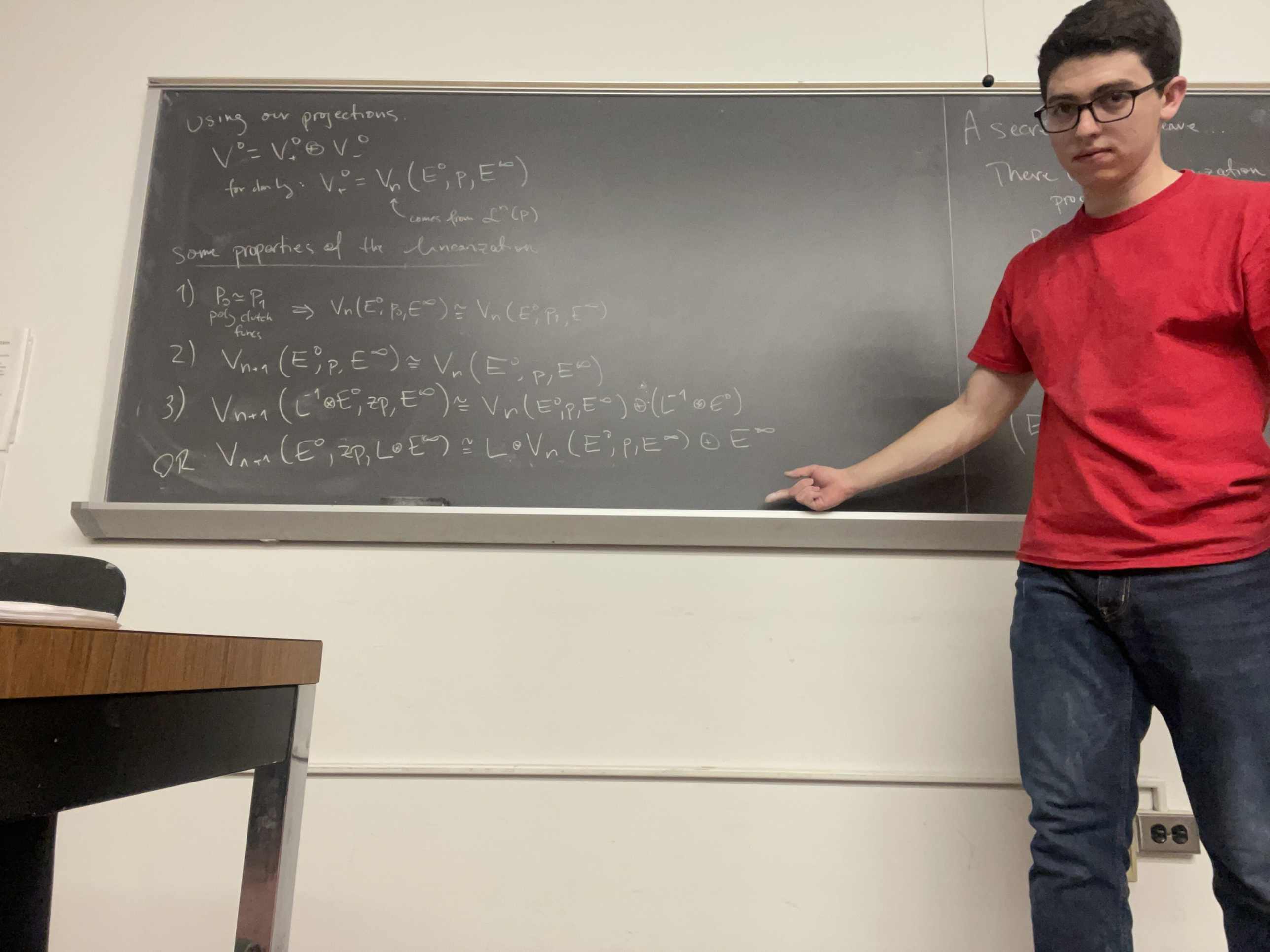
linear in $z$

Prop

$(E^\circ, P, E^\infty) \oplus (\sum_{k=1}^{n} L^k \otimes E^\circ,$

$\cong (\sum_{k=0}^{n} L^k \otimes E^\circ,$

$= V^\circ$


Using our projections.
$V^0 = V^0_+ \oplus V^0_-$
for clarity: $V^0_+ = V_n(E^0, p, E^\infty)$
comes from $\mathcal{L}^n(p)$
Some properties of the linearization
1) $p_0 \simeq p_1$ poly clutch funcs $\Rightarrow V_n(E^0, p_0, E^\infty) \cong V_n(E^0, p_1, E^\infty)$
2) $V_{n+1}(E^0, p, E^\infty) \cong V_n(E^0, p, E^\infty)$
3) $V_{n+1}(L^{-1} \otimes E^0, zp, E^\infty) \cong V_n(E^0, p, E^\infty) \oplus (L^{-1} \otimes E^0)$
OR $V_{n+1}(E^0, zp, L \otimes E^\infty) \cong L \otimes V_n(E^0, p, E^\infty) \oplus E^\infty$

Home Stretch!

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Home Stretch!

Thm: $\mu : K(x)[t]/(t-1)([L]t-1) \longrightarrow K(P)$

$t \longmapsto [H]$

is an iso

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Stretch!

$\mu : K(x)[t]/(t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$

remember, H is the dual of $\Pi^*$

is an iso

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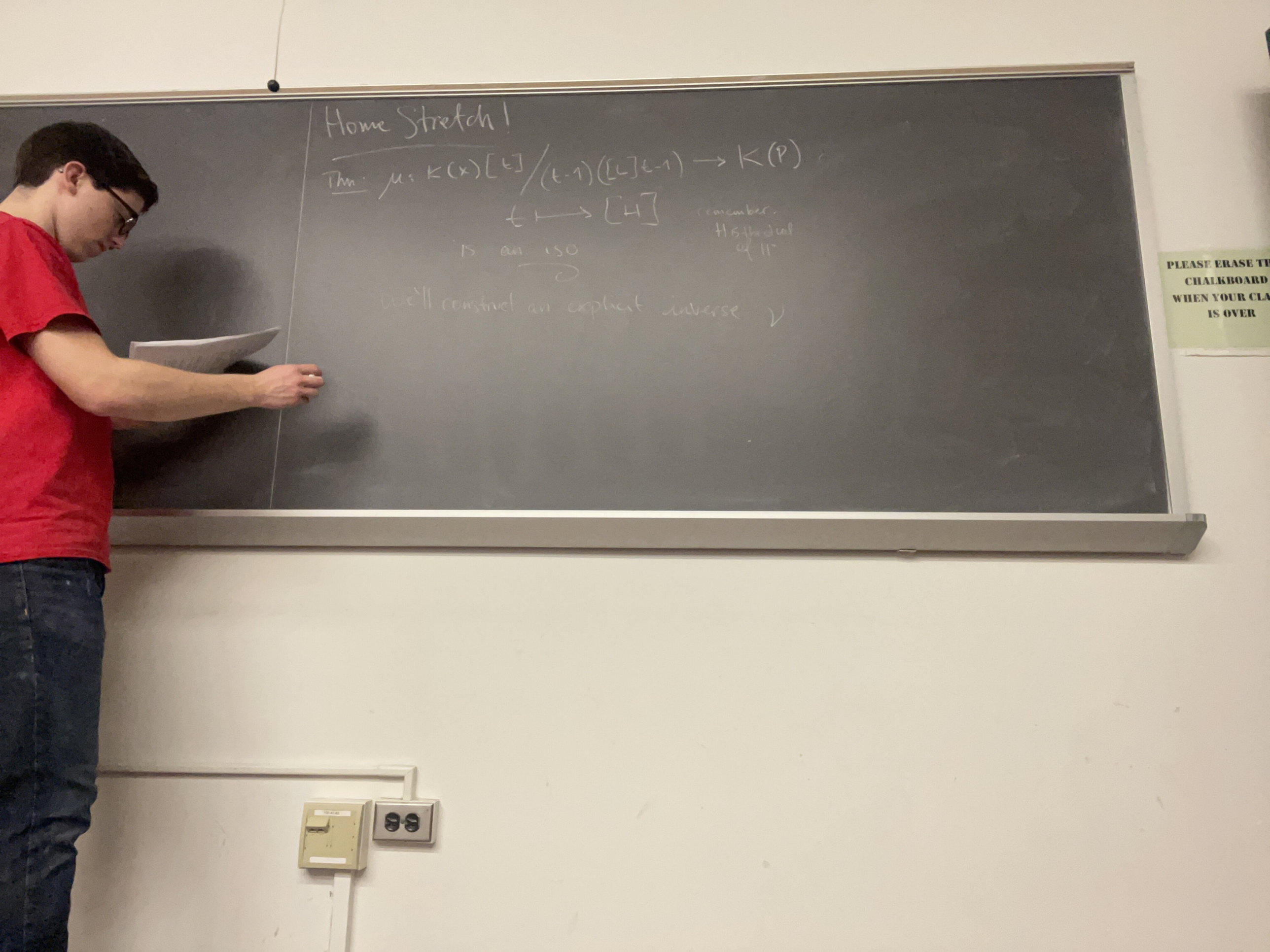
Home Stretch!
Thm: $\mu : K(X)[t] / (t-1)([L]t-1) \to K(P)$
$t \longmapsto [H]$
remember: H is the dual of H
is an iso
we'll construct an explicit inverse $\nu$
PLEASE ERASE TH
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WHEN YOUR CLA
IS OVER

# Home Stretch!

Thm: $\mu : K(x)[t]/(t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$ remember, H is the dual of H'

is an iso

We'll construct an explicit inverse ↓

f clutches $(E^\circ; E^\infty)$

$\otimes E^\circ)$

$\oplus E^\infty$

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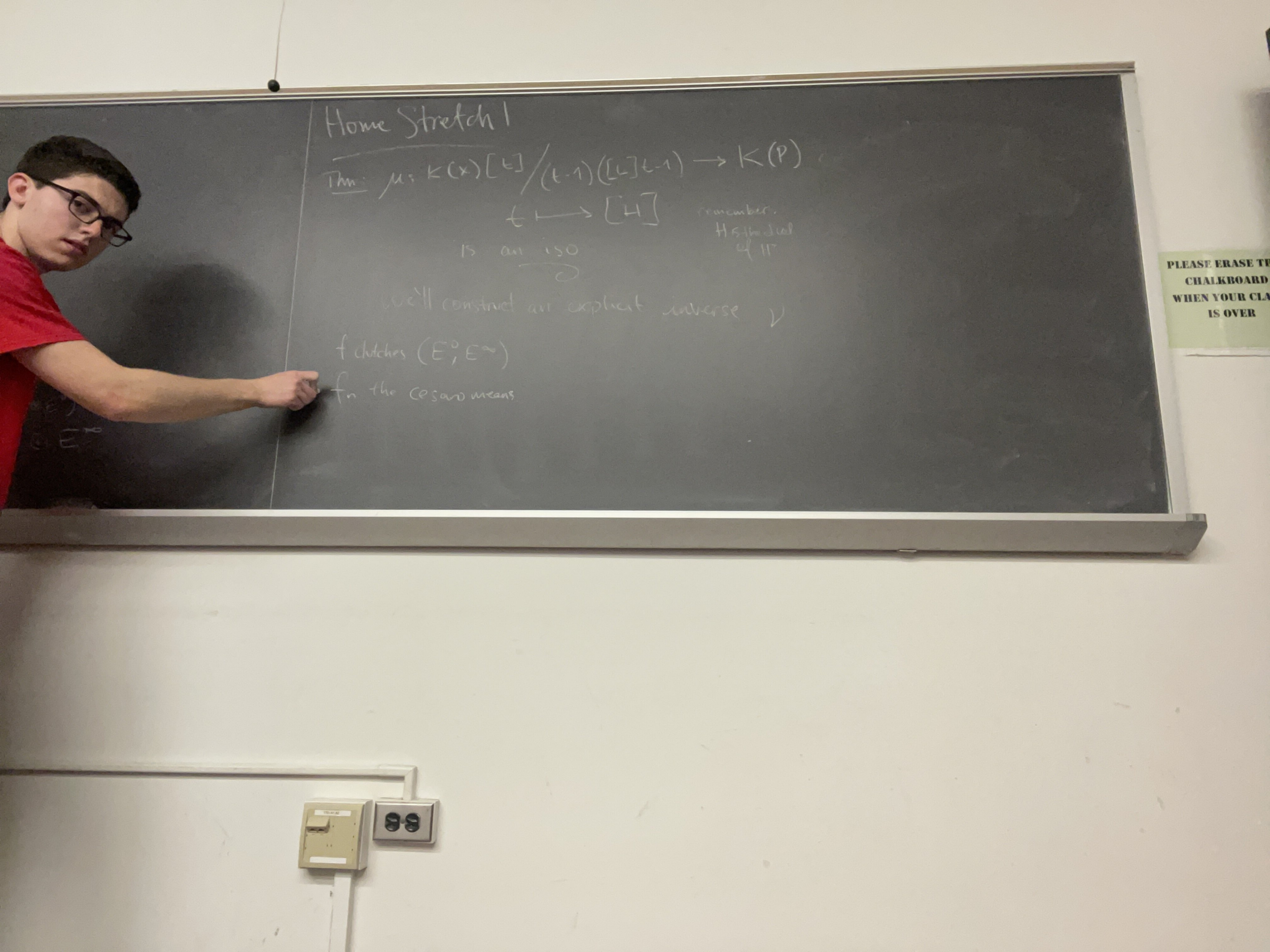
IS OVER


Home Stretch 1
Thm: μ: K(X)[t]/(t-1)([L]t-1) → K(P)
t ⟼ [H]
remember: H is the dual of H
is an iso
we'll construct an explicit inverse ν
f clutches (E°, E∞)
fn the cesaro means
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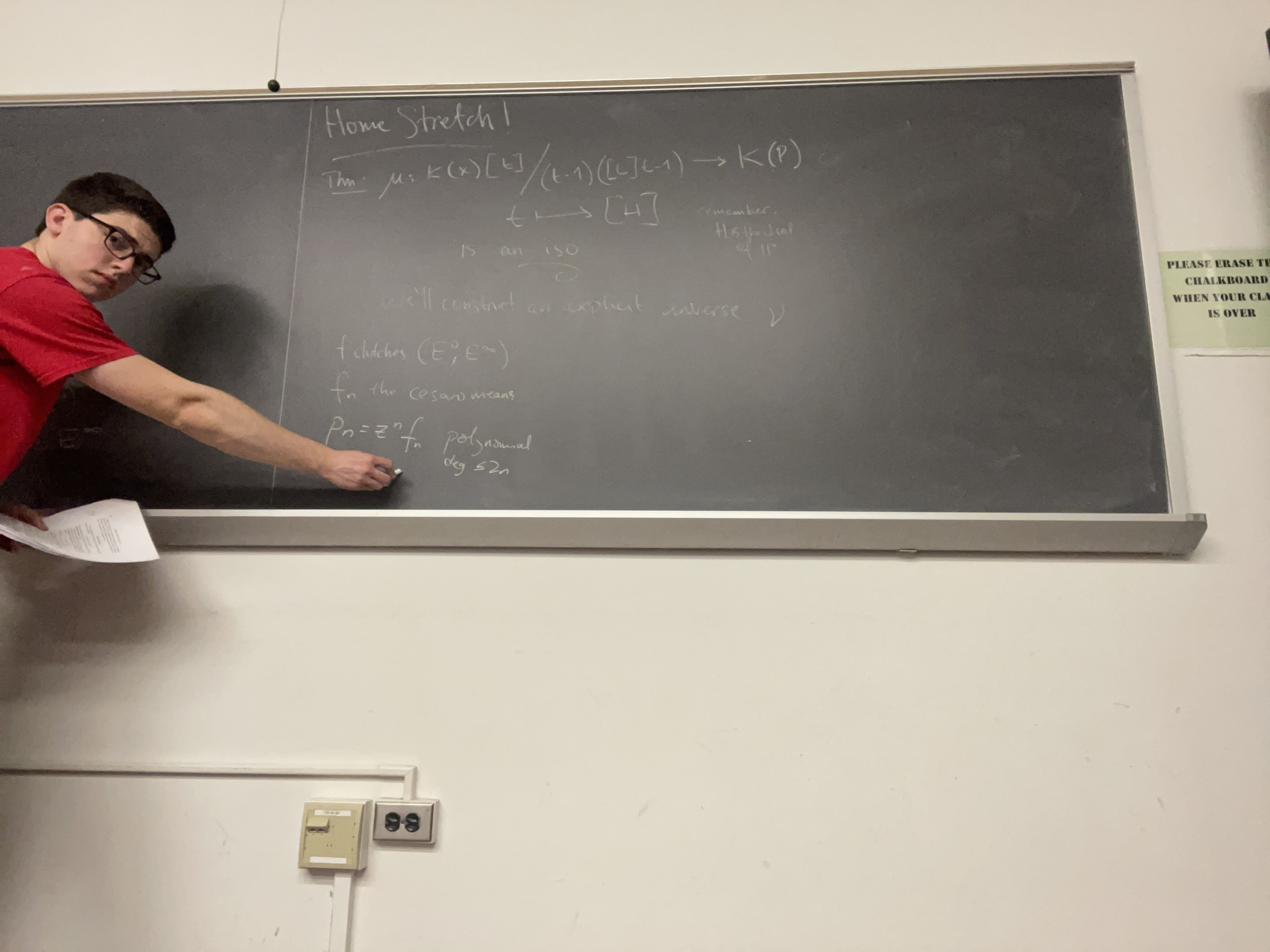
IS OVER


Home Stretch!
Thm: $\mu : K(X)[t] / (t-1)([L]t-1) \to K(P)$
$t \longmapsto [H]$
remember, H is the dual of $\mathbb{P}$
is an iso
we'll construct an explicit inverse $\nu$
f clutches $(E^0, E^\infty)$
$f_n$ the cesaro means
$p_n = z^n f_n$ polynomial
deg $\leq 2n$
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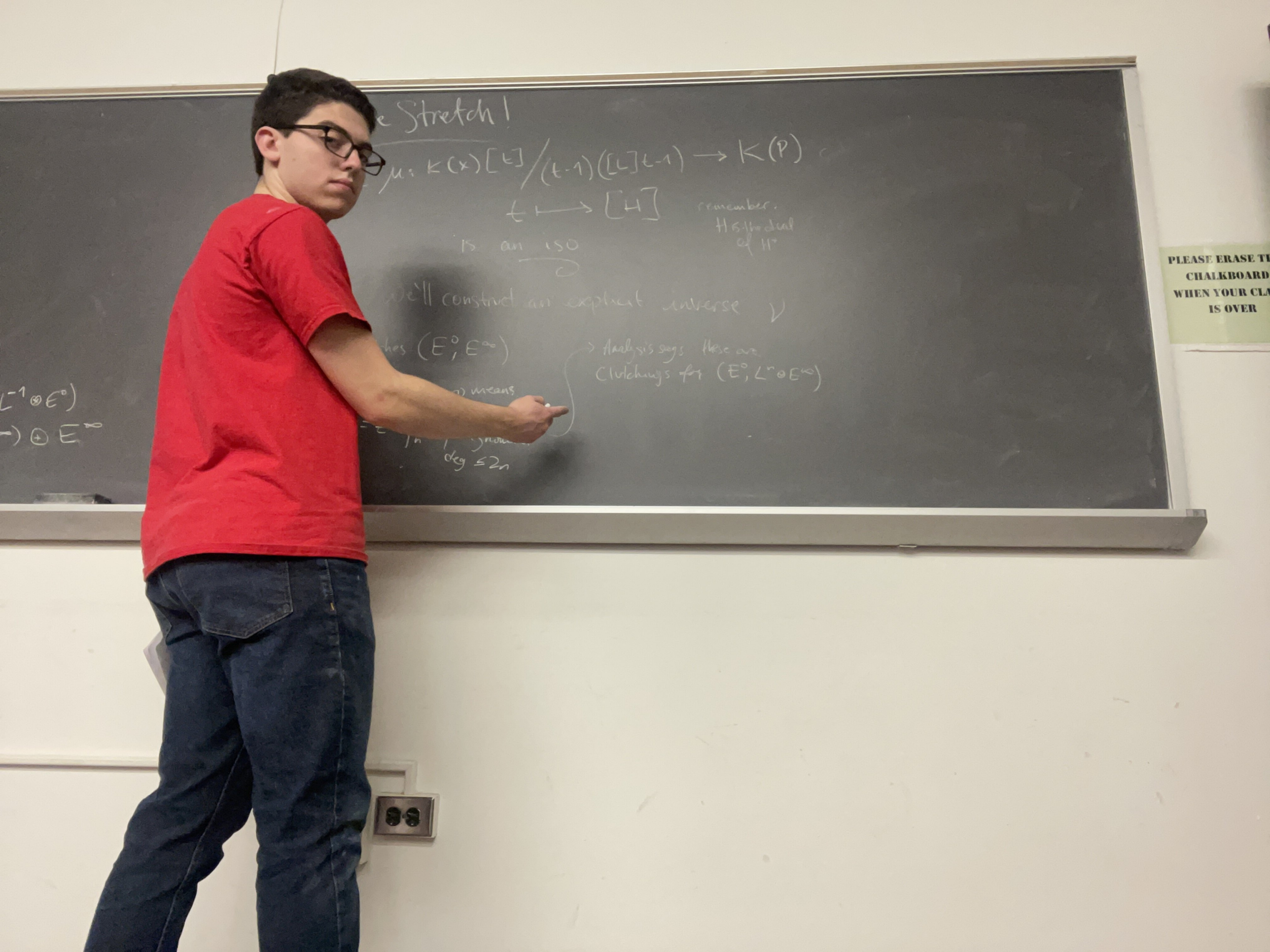
IS OVER


Stretch!
μ: K(x)[t]/(t-1)([L]t-1) → K(P)
t ⟼ [H]
remember.
H is the dual of H°
is an iso
We'll construct an explicit inverse ν
(E°, E∞)
Analysis says these are
Clutchings for (E°, L^n ⊗ E∞)
means
deg ≤ 2n
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IS OVER

$L^{-1} \otimes E^\circ)$

$\to) \oplus E^\infty$

# Home Stretch I

**Thm:** $\mu : K(x)[t] \Big/ (t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$

remember: $H$ is the dual of $H'$

is an iso

we'll construct an explicit inverse $\nu$

f clutches $(E^\circ, E^\infty)$

fn the cesaro means.

$P_n = z^n f_n$ polynomial deg $\leq 2n$

→ Analysis says these are Clutchings for $(E^\circ, L^n \otimes E^\infty)$

↖ comes from $z^n$

Home Stretch!

Thm: $\mu : K(x)[t]/(t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$

remember: H is the dual of H'

A is an iso

we'll construct an explicit inverse

f clutches $(E^\circ, E^\infty)$

fn the cesaro means

$P_n = z^n f_n$ polynomial deg $\leq 2n$

→ Analysis says these are Clutchings for $(E^\circ, L^n \oplus E^\infty)$

↖ comes from $z^n$

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IS OVER

define $\nu_n(f) \in A$

$= [V_{2n}(E^0, \rho_n, L'' \otimes E^\infty)]$

Home Stretch

Thm: $\mu : K(X)$

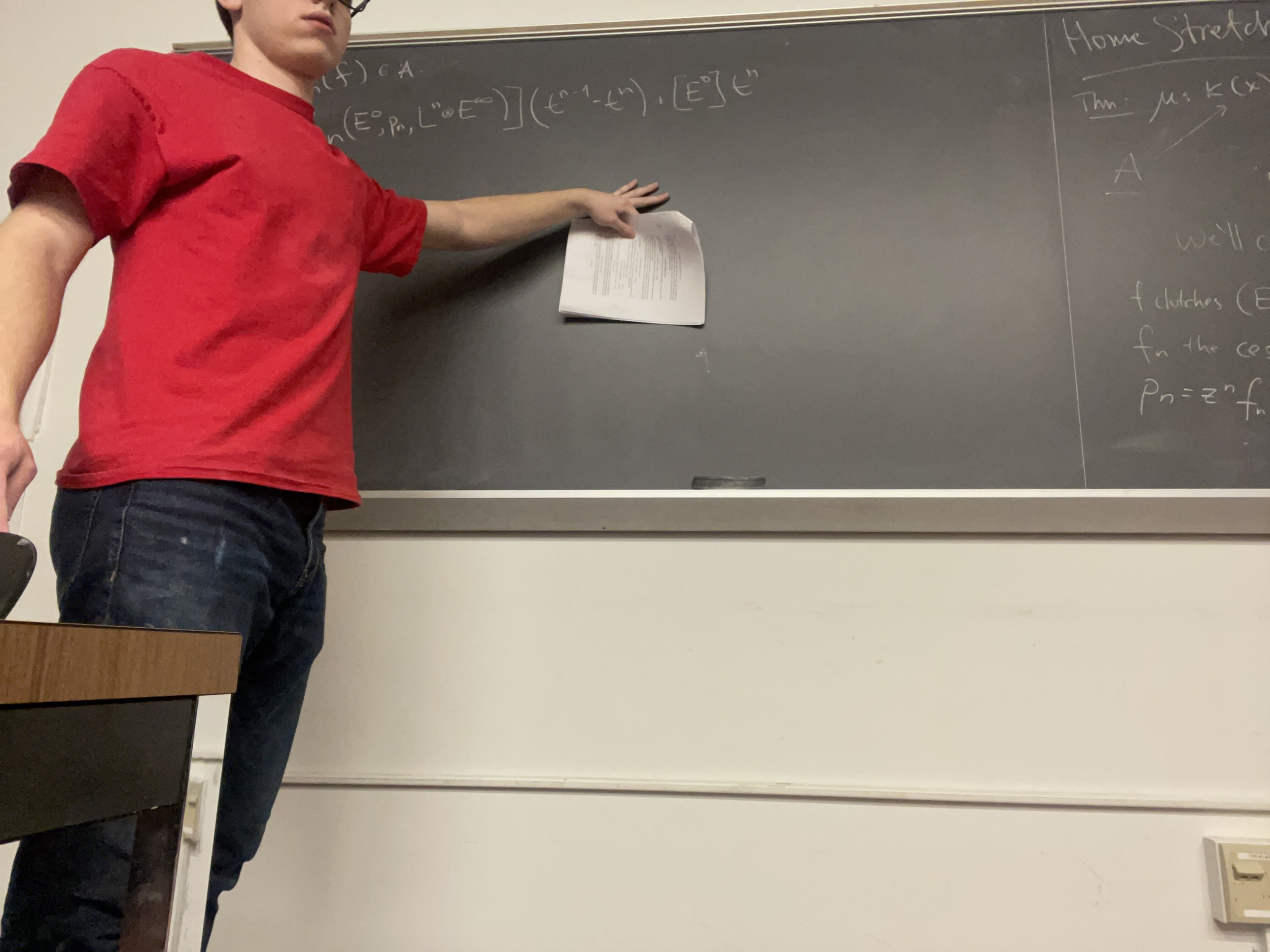
$A$

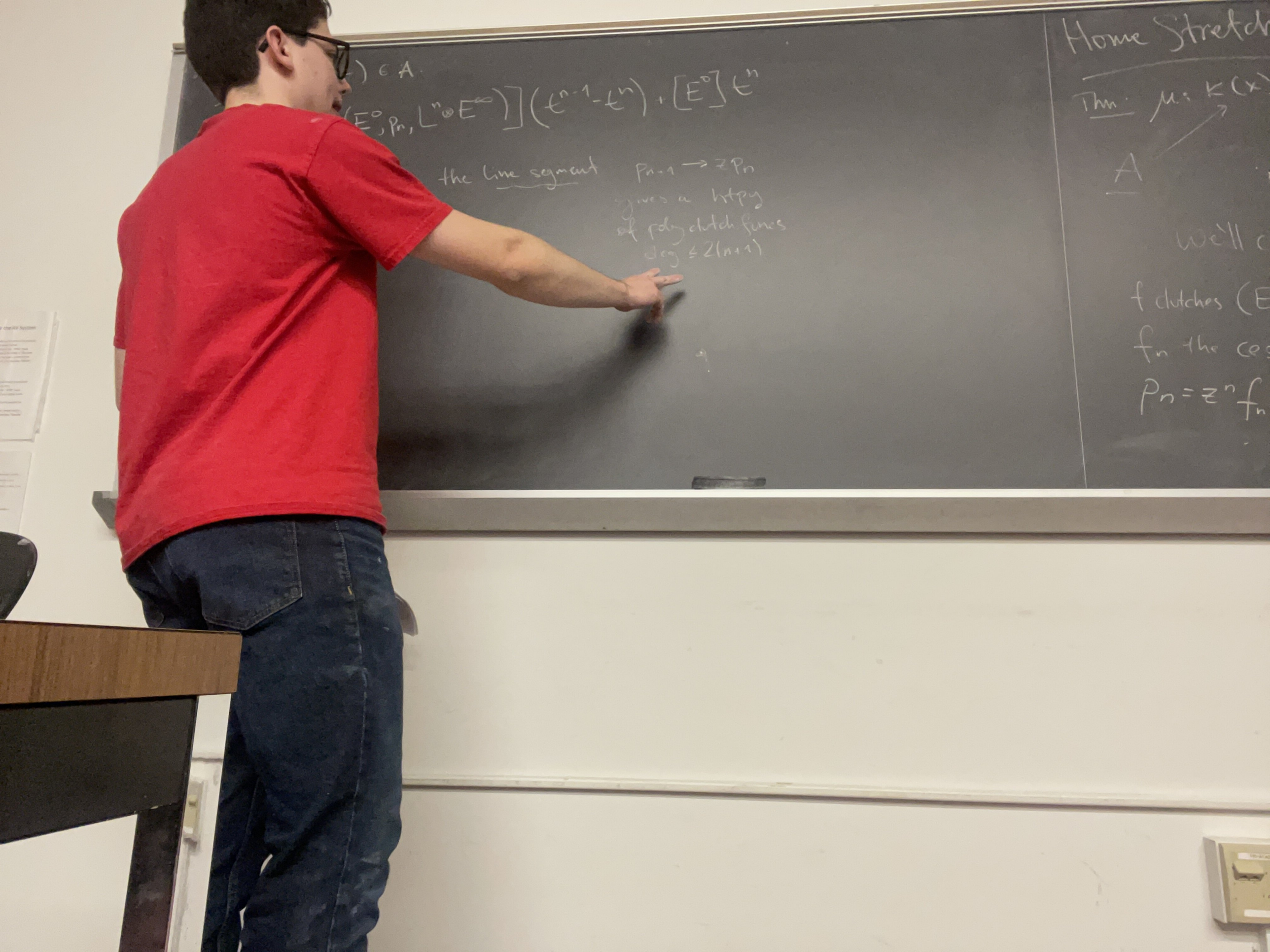
we'll

f clutches (E

$f_n$ the ce

$\rho_n = z^n f_n$


Home Stretch
Thm: μ: K(x)
A
We'll c
f clutches (E
fn the ces
Pn = z^n fn



$) \in A$
$(E^0, p_n, L^n \otimes E^\infty)](t^{n-1} - t^n) + [E^0]\, t^n$
the line segment $p_{n+1} \to z p_n$
gives a htpy
of poly clutch funcs
deg $\leq 2(n+1)$
Home Stretch
Thm: $\mu : K(x)$
A
We'll c
f clutches (E
$f_n$ the ces
$p_n = z^n f_n$

define $v_n(f) \in A$.

$$= [V_{2n}(E^\circ, P_n, L^n \otimes E^\infty)]\,(t^{n-1} - t^n) \cdot [E^\circ]\, t^n$$

for large enough $n$ the line segment $P_{n-1} \to zP_n$ gives a htpy of poly clutch funcs deg $\leq 2(n+1)$

by properties from before:

$$V_{2n+2}(E^\circ, P_{n+1}, L^{n+1} \otimes E^\infty)$$

Home Stretch

Thm: $\mu : K(x$

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$V_{2n+2}(E^\circ, zP_n, L^{n+1} \otimes E^\infty)$

Home Stretch

Thm $\mu: K(x)$

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$V_{2n+2}(E^\circ, P_{n+1}, L^{n+1} \otimes E^\infty)$

$\cong V_{2n+2}(E^\circ, zP_n, L^{n+1} \otimes E^\infty)$

$= V_{2n+1}(E^\circ, zP_n, L^{n+1} \otimes E^\infty)$

Home Stretch

Thm: $\mu: K(x)$

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by properties from before

$$V_{2n+2}(E^\circ, p_{n+1}, L^{n+1} \otimes E^\infty)$$
$$\cong V_{2n+2}(E^\circ, z p_n, L^{n+1} \otimes E^\infty)$$
$$\cong V_{2n+1}(E^\circ, z p_n, L^{n+1} \otimes E^\infty)$$
$$\cong L \otimes V_{2n}(E^\circ, p_n, L^n \otimes E^\infty) \oplus E^\circ$$

Home Stretch

Thm: $\mu: K(X$

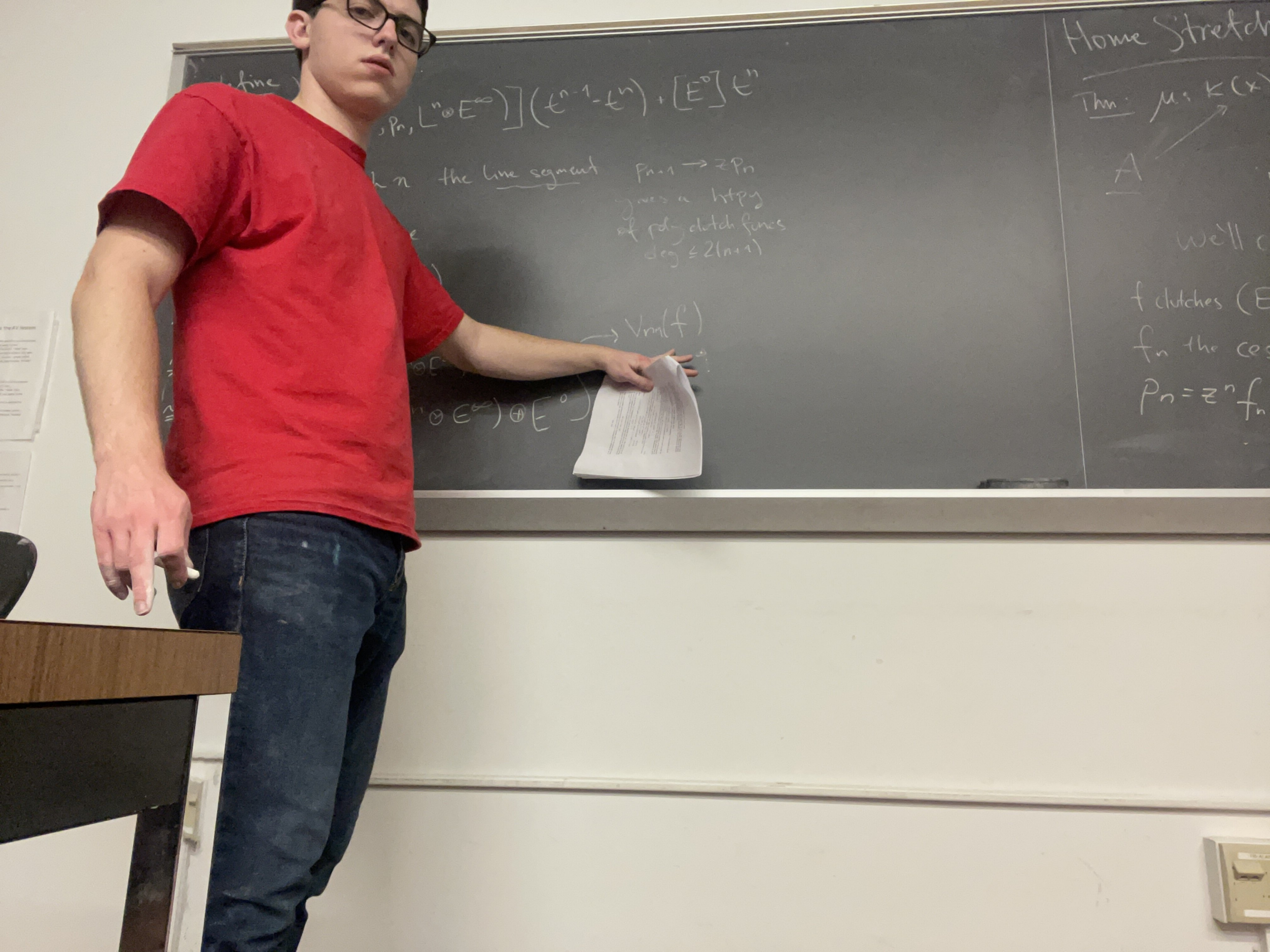
$A$

we'll

f clutches (E

$f_n$ the ce

$p_n = z^n f_n$


the line segment
gives a htpy
of poly clutch funcs
deg ≤ 2(n+1)
Home Stretch
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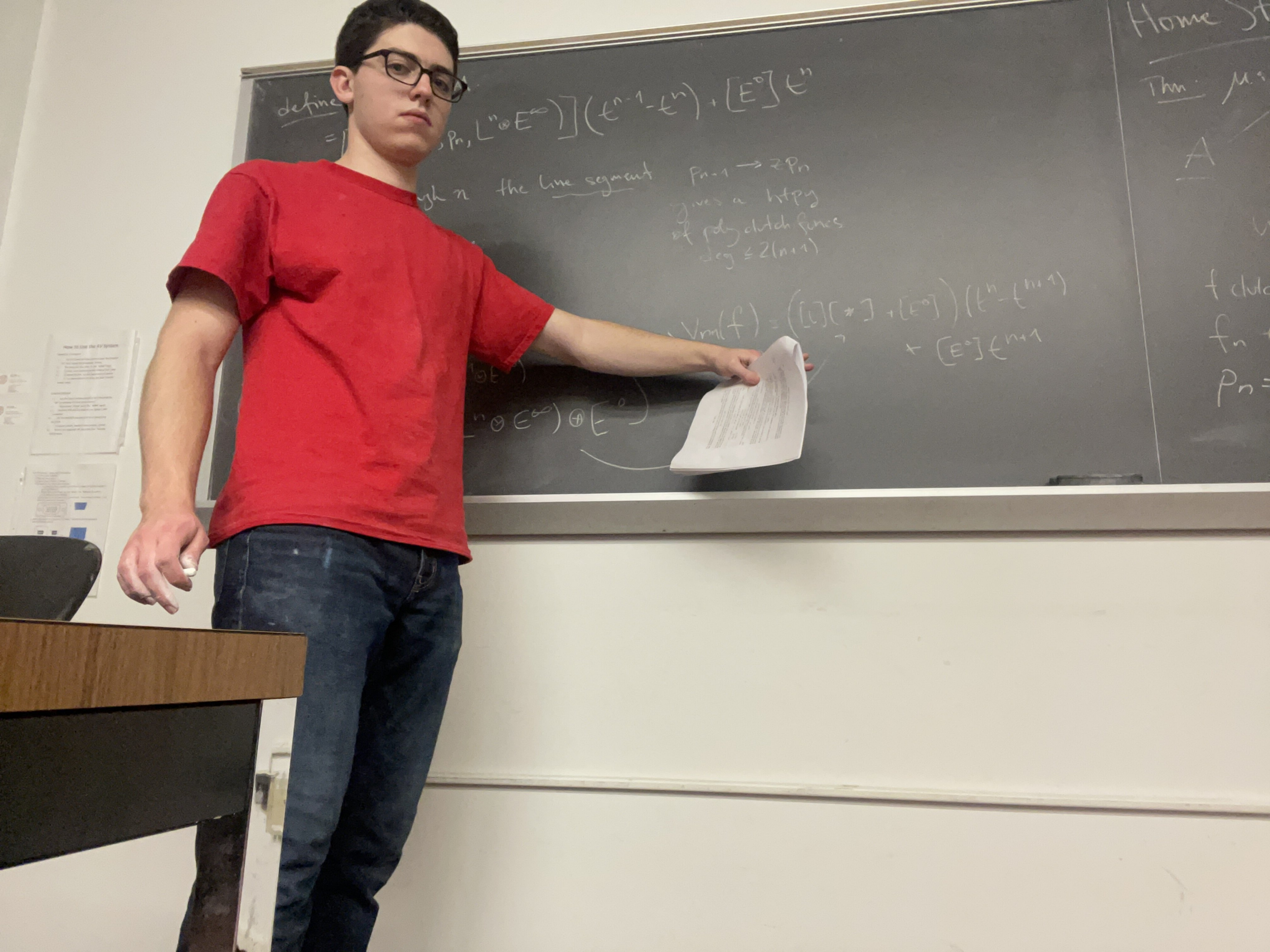
define
the line segment
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deg ≤ 2(n+1)
Home St
Thm:
A

define $V_n(f) \in A$

$= [V_{2n}(E^\circ, P_n, L^n \otimes E^\infty)](t^{n-1} - t^n) + [E^\circ] t^n$

for large enough $n$ the line segment $P_{n-1} \to \mathbb{Z}P_n$ gives a htpy of poly clutch funcs deg $\leq 2(n+1)$

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$V_{2n+2}(E^\circ, P_{n+1}, L^{n+1} \otimes E^\infty)$

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$\to V_{n+1}(f) = ([L][*] + [E^\circ])(t^n - t^{n+1}) + [E^\circ] t^{n+1}$

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Home St

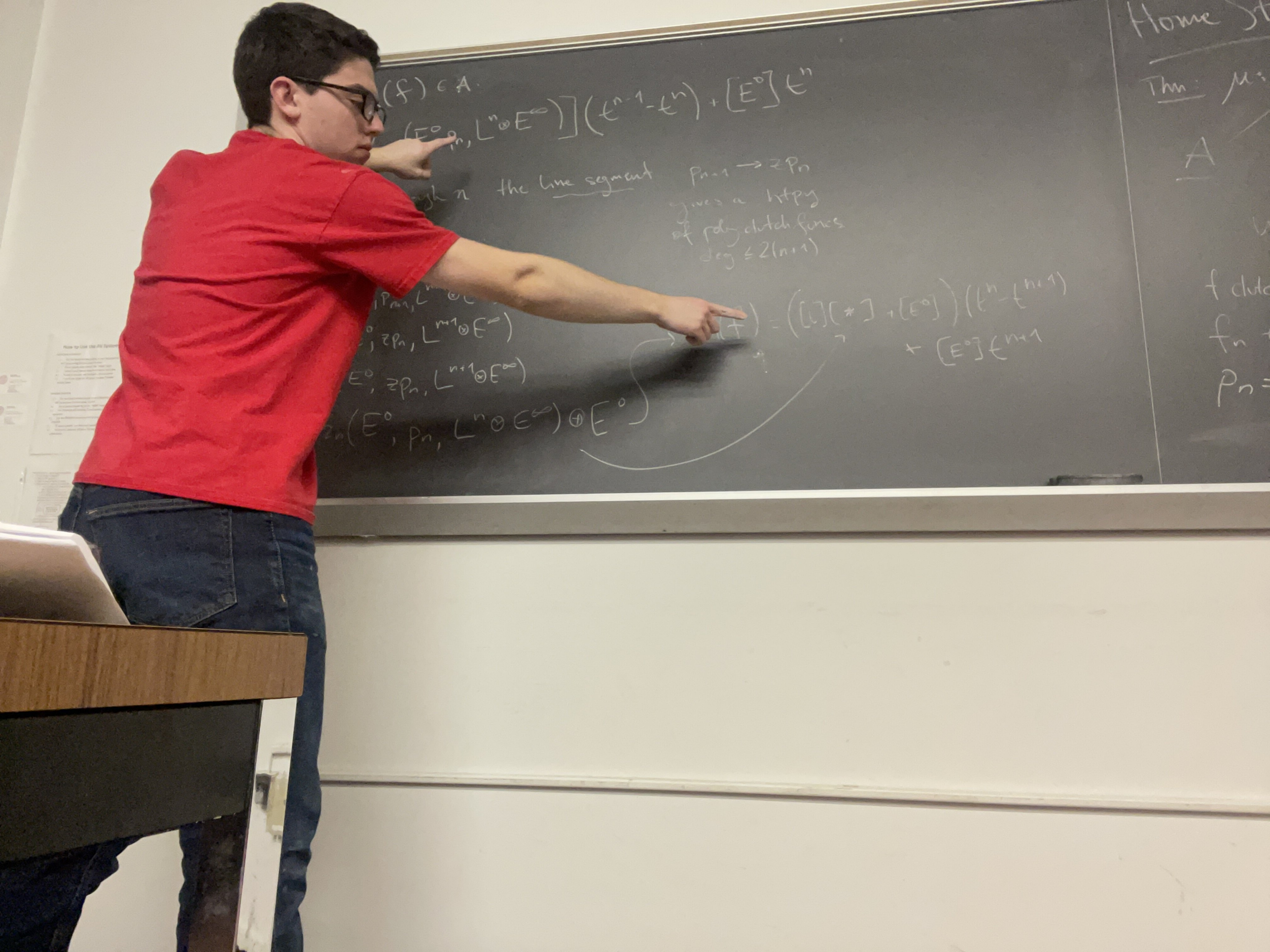
Thm: M

A

f clut

$f_n$

$P_n$ =


(f) ∈ A.
the line segment
p_{n+1} → z p_n
gives a htpy
of poly clutch funcs
deg ≤ 2(n+1)
+ [E^0] t^{n+1}
Home St
Thm:
A
f clut
f_n
p_n =

Define $V_n(f) \in A$.

$$= \left[ V_{2n}(E^\circ, P_n, L^n \otimes E^\infty) \right] (t^{n-1} - t^n) + [E^\circ] t^n$$

for large enough $n$ the line segment $P_{n-1} \to \Sigma P_n$ gives a htpy of pd clutch fonces deg $\leq 2(n+1)$

by properties from before

$$V_{2n+2}(E^\circ, P_{n+1}, L^{n+1} \otimes E^\infty)$$
$$\cong V_{2n+2}(E^\circ, \Sigma P_n, L^{n+1} \otimes E^\infty)$$
$$\cong V_{2n+1}(E^\circ, \Sigma P_n, L^{n+1} \otimes E^\infty)$$
$$\cong L \otimes V_{2n}(E^\circ, P_n, L^n \otimes E^\infty) \oplus E^\circ$$

$$\to V_{n+1}(f) = \left( [1][*] + [E^\circ] \right)(t^n - t^{n+1}) + [E^\circ] t^{n+1}$$

Home St

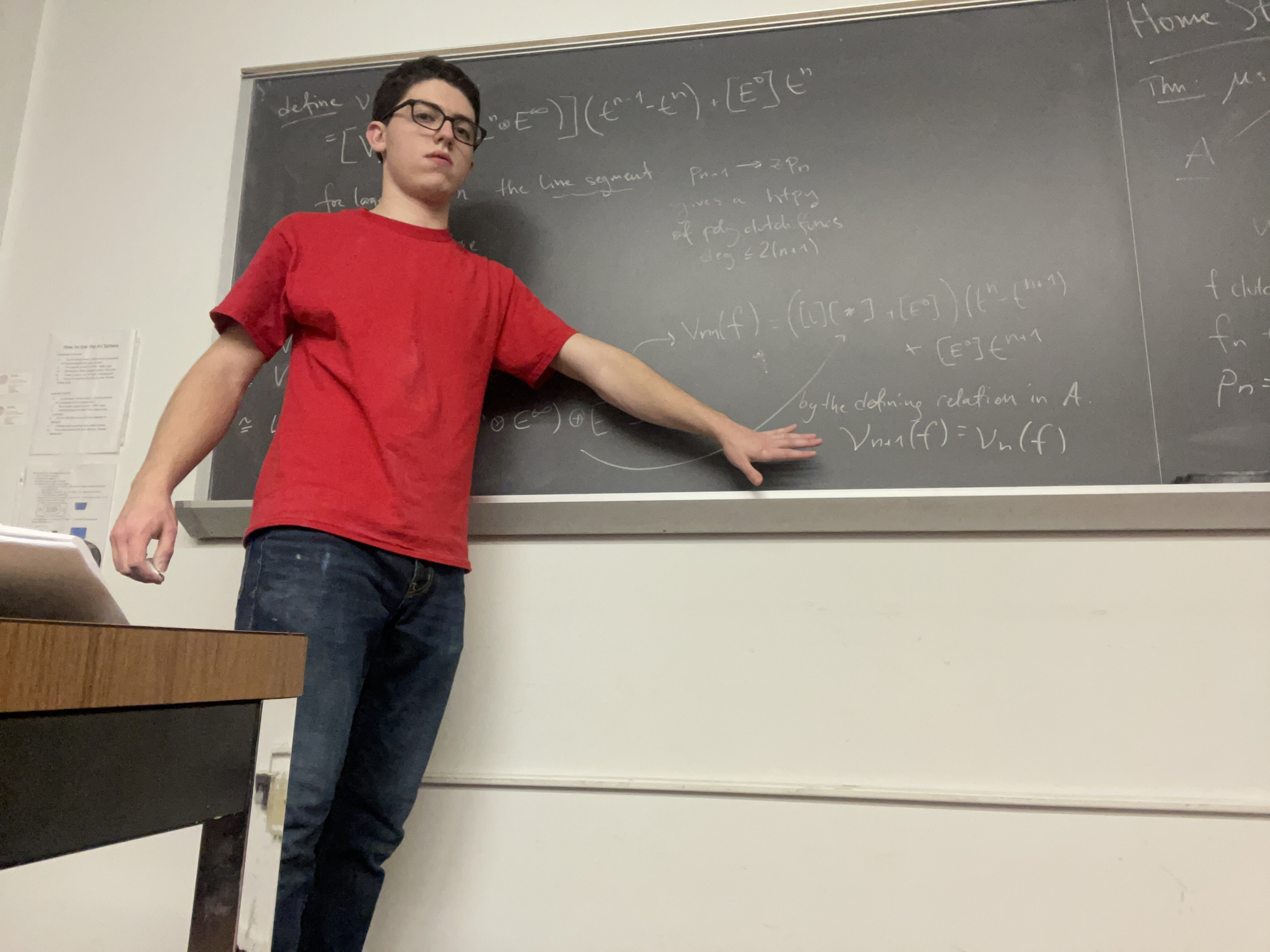
Thm: M

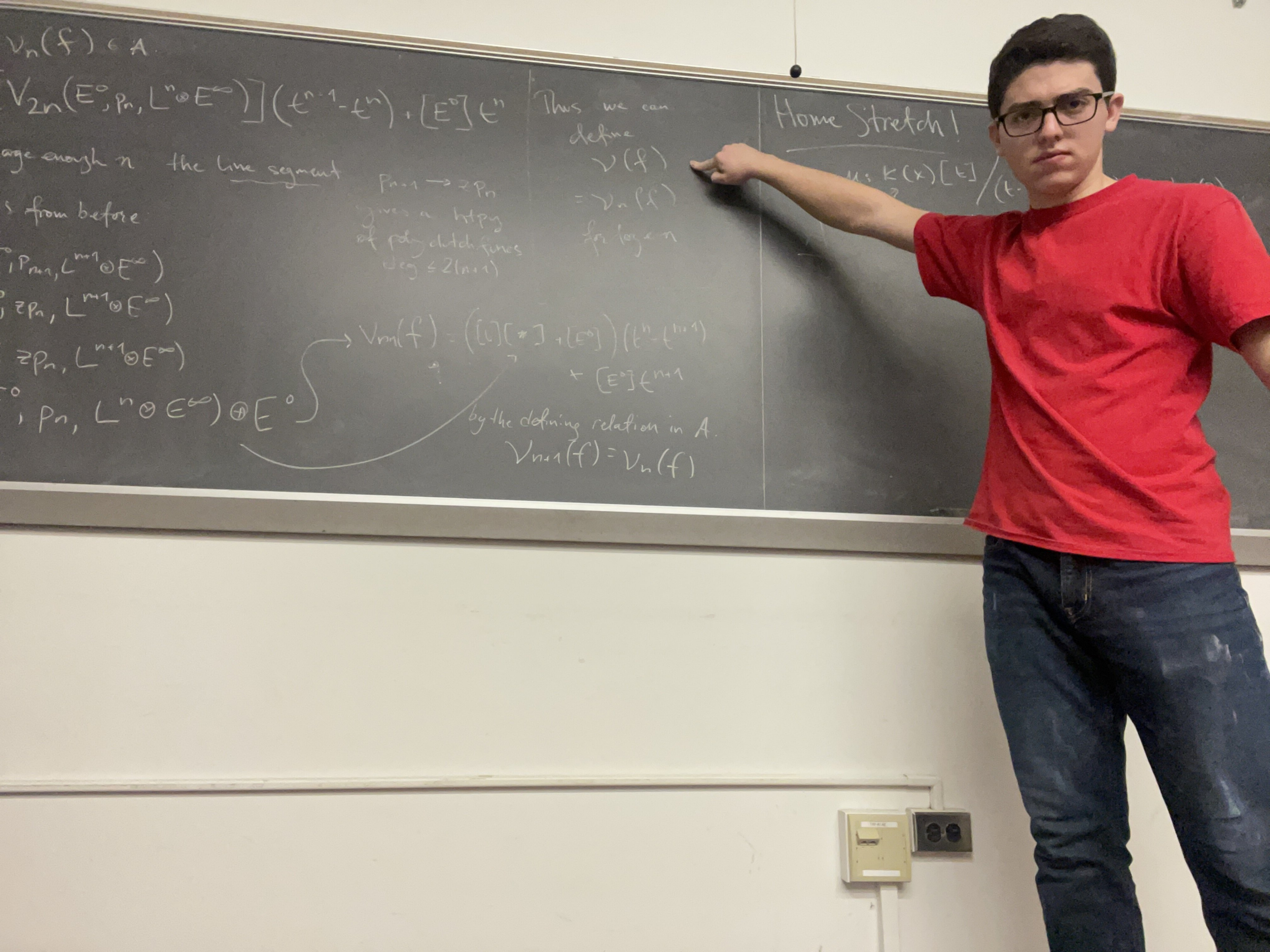
A

f clut

$f_n$

$P_n$ =


define
the line segment
gives a htpy
of poly clutch funcs
deg ≤ 2(n+1)
by the defining relation in A.
Home St
Thm:



$\nu_n(f) \in A$
$V_{2n}(E^0, p_n, L^n \otimes E^\infty)](t^{n-1} - t^n) + [E^0]t^n$
age enough $n$ the line segment
s from before
$p_{n+1} \to z p_n$
gives a htpy
of poly clutch funcs
deg $\leq 2(n+1)$
$p_{n+1}, L^{n+1} \otimes E^\infty)$
$z p_n, L^{n+1} \otimes E^\infty)$
$z p_n, L^{n+1} \otimes E^\infty)$
$p_n, L^n \otimes E^\infty) \oplus E^0$
$\nu_{n+1}(f) = ([t][*] + [E^0])(t^n - t^{n+1})$
$+ [E^0]t^{n+1}$
by the defining relation in $A$.
$\nu_{n+1}(f) = \nu_n(f)$
Thus we can
define
$\nu(f)$
$= \nu_n(f)$
for large $n$
Home Stretch!
$K(x)[t]$

$]\,(t^{n-1}-t^n) + [E^0]\,t^n$

segment

$P_{n-1} \to z P_n$ gives a htpy of poly clutch funcs deg $\leq 2(n+1)$

$\to V_{n+1}(f) = \left([L][*] + [E^0]\right)(t^n - t^{n+1}) + [E^0]\,t^{n+1}$

by the defining relation in $A$.

$V_{n+1}(f) = V_n(f)$

Thus we can define $\nu(f) = \nu_n(f)$ for large $n$

# Home Stretch!

Thm: $\mu : K(X)[t] / (t-1)([L]t-1) \to K(P)$

$A$

$t \longmapsto [H]$ is an iso

remember: $H$ is the dual of $H^*$

Because iso classes of bundles in $K(P)$ depend on htpy classes of clutching funcs we can define $\nu(E)$ as the inverse map

$(t^{n-1}-t^n)\cdot[E^0]t^n$

segment

$P_{n-1} \to \Sigma P_n$

gives a htpy

of poly clutch f

deg ≤ 2(n+

$\to Vect(f)$

Thus we can

define

$\nu($

$= \nu_n$

Home Stretch!

Thm: $\mu: K(X)[t]/(t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$

remember, H is the dual of H*

A

is an iso

Because iso classes of bundles in $K(P)$ depend on htpy classes of clutching funcs we can define $\nu(E)$ as the inverse map.

It's a $K(X)$ module hom,

$(t^{n-1}-t^n)\cdot[E^0]t^n$

segment $P_{n-1} \to \Sigma P_n$

gives a htpy of poly clutch. funcs deg $\leq 2(n+1)$

$\to V_{nn}(f) = ([1][*] + [E^0])(t^n - t^{n+1})$

$+ [E^0]t^{n+1}$

$0$

by the defining relation in $A$.

$V_{n+1}(f) = V_n(f)$

Thus we can define

$V(f)$

$= V_n(f)$

for large $n$

## Home Stretch!

Thm: $\mu : K(X)[t] / (t-1)([L]t-1) \to K(P)$

$t \longmapsto [H]$

$A$

is an iso

remember $H$

Because iso classes of bundles depend on htpy classes of clu

we can define $V(E)$ as t

Its a $K(X)$ module hom, (Exercise)

ma

$$\mu_{V}[E] = \mu\{[V_{2n}(E^{\circ}, P_n, L^n \oplus E^{\infty})](t^{n-1} - t^n) + [E^{\circ}]t^n\}$$

$$= [V_{2n}(E^{\circ}, P_n, L^n \oplus E^{\infty})]([H]^{n-1} - [H]^n) + [E^{\circ}][H]^n$$

$$= [E^{\circ}, P_n, L^n \oplus E^{\infty}][H]^n$$

$$= [E^{\circ}, f_n, E^{\infty}]$$

$$= [E^{\circ}, f, E^{\infty}]$$

$$= [E]$$

Home Stretch

Thm: $\mu: K(x)$

A

Because

depe

NC

It's a

$$\mu_{VLE} = \mu\{[V_{2n}(E^\circ, P_n, L^n \otimes E^\omega)](t^{n-1} - t^n) + [E^\circ]t^n\}$$

$$= [V_{2n}(E^\circ, P_n, L^n \otimes E^\omega)]([H]^{n-1} - [H]^n) + [E^\circ][H]^n$$

$$= [E^\circ, P_n, L^n \otimes \quad ][H]^n$$

$$= [E^\circ \quad E^\omega$$

$$= [$$

$$= [$$

Apparently this equality means something

Home Stretch

Thm: $\mu$: E(x)

A

Because

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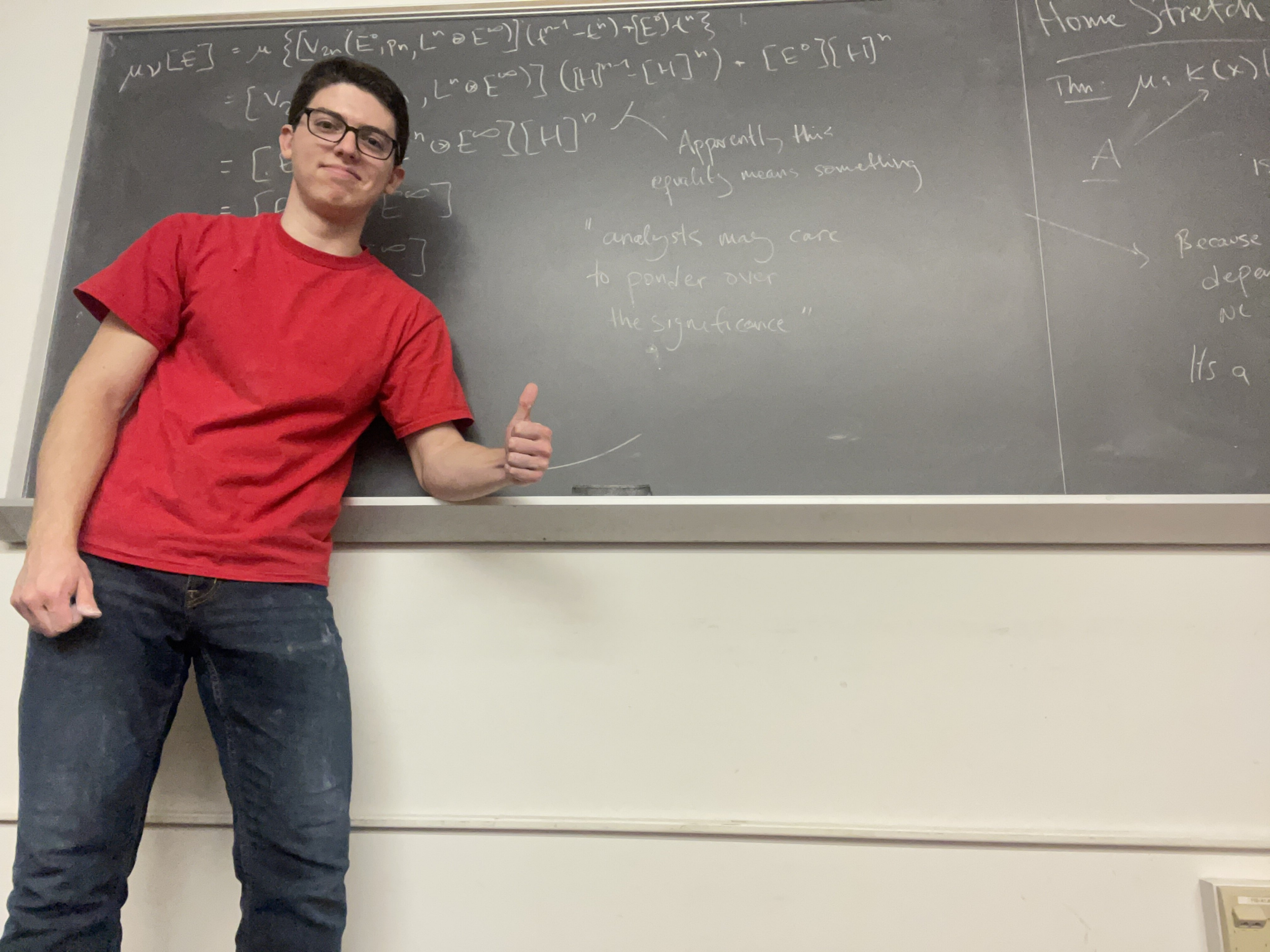
Hs 9


Apparently this
equality means something
"analysts may care
to ponder over
the significance"
Home Stretch
Thm:
A
Because
Its a

$$\{[V_{2n}(E^\circ, P_n, L^n \otimes E^{\infty})](t^{n-1} - t^n) + [E^\circ]t^n\}$$

$$[V_{2n}(E^\circ, P_n, L^n \otimes E^{\infty})]([H]^{n-1} - [H]^n) + [E^\circ][H]^n$$

$$= [E^\circ, P_n, L^n \otimes E^{\infty}][H]^n$$

$$= [E^\circ, f_n, E^{\infty}]$$

$$[E^\circ, f, E^{\infty}]$$

$$[E]$$

Home Stretch

Thm: $\mu: K(X)$

A

Because

depe

NC

Hs 9

$$\mu\nu[E] = \mu\{[V_{2n}(E^\circ, P_n, L^n \otimes E^{\infty})](t^{n-1} - t^n) + [E^\circ]t^n\}$$

$$= [V_{2n}(E^\circ, P_n, L^n \otimes E^{\infty})]([H]^{n-1} - [H]^n) + [E^\circ][H]^n$$

$$= [E^\circ, P_n, L^n \otimes E^{\infty}][H]^n$$

$$= [E^\circ, f_n, E^{\infty}]$$

$$= [E^\circ, f, E^{\infty}]$$

$$= [E]$$

$$\nu\mu(t^n) = \nu[H^n]$$

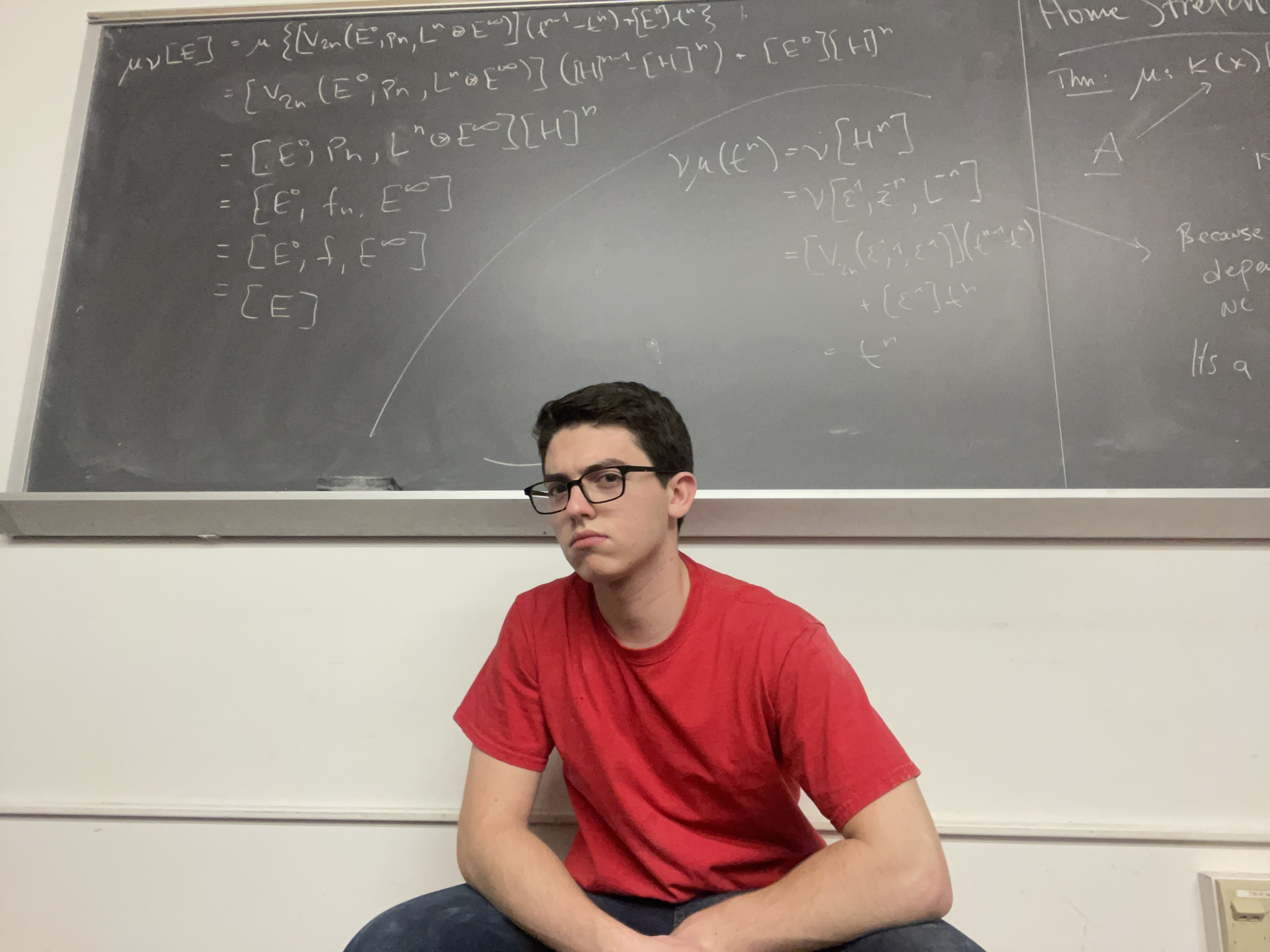
$$= \nu[\varepsilon^1, z^n, L^{-n}]$$

$$= [V_{2n}(\varepsilon^1, 1, \varepsilon^1)](t^{n-1} - t^n) + [\varepsilon^1]t^n$$

$$= t^n$$

Home Sta

Thm:



$\mu\nu[E] = \mu\{[V_{2n}(E^0, p_n, L^n \otimes E^\infty)](t^{n-1} - t^n) + [E^0]t^n\}$
$= [V_{2n}(E^0, p_n, L^n \otimes E^\infty)]([H]^{n-1} - [H]^n) + [E^0][H]^n$
$= [E^0, p_n, L^n \otimes E^\infty][H]^n$
$= [E^0, f_n, E^\infty]$
$= [E^0, f, E^\infty]$
$= [E]$
$\nu\mu(t^n) = \nu[H^n]$
$= \nu[\varepsilon^1, z^{-n}, L^{-n}]$
$= [V_{2n}(\varepsilon^1, 1, \varepsilon^1)](t^{n-1} - t^n)$
$+ [\varepsilon^1]t^n$
$= t^n$
Home Stretch
Thm: $\mu : K(X)$
A
Because
depe
NC
Its a

Linearization:

Linearization:

$$P = \sum_{k=0}^{n} a_k z^k$$

# Linearization:

$$P = \sum_{k=0}^{n} a_k z^k$$

clutching func
for $(E^0, E^\infty)$

Linearization:

$\mathcal{L}^n(p) : \pi^*\left(\Sigma_{k=0}^{n} L^k \otimes E^q\right) \longrightarrow \pi^*\left(E^{\infty} \otimes \sum_{k=1}^{n} L^k \otimes E^0\right)$

clutching
for $(E^0, E^q)$

Linearization:

$P = \sum_{k=0}^{n}$ [illegible]

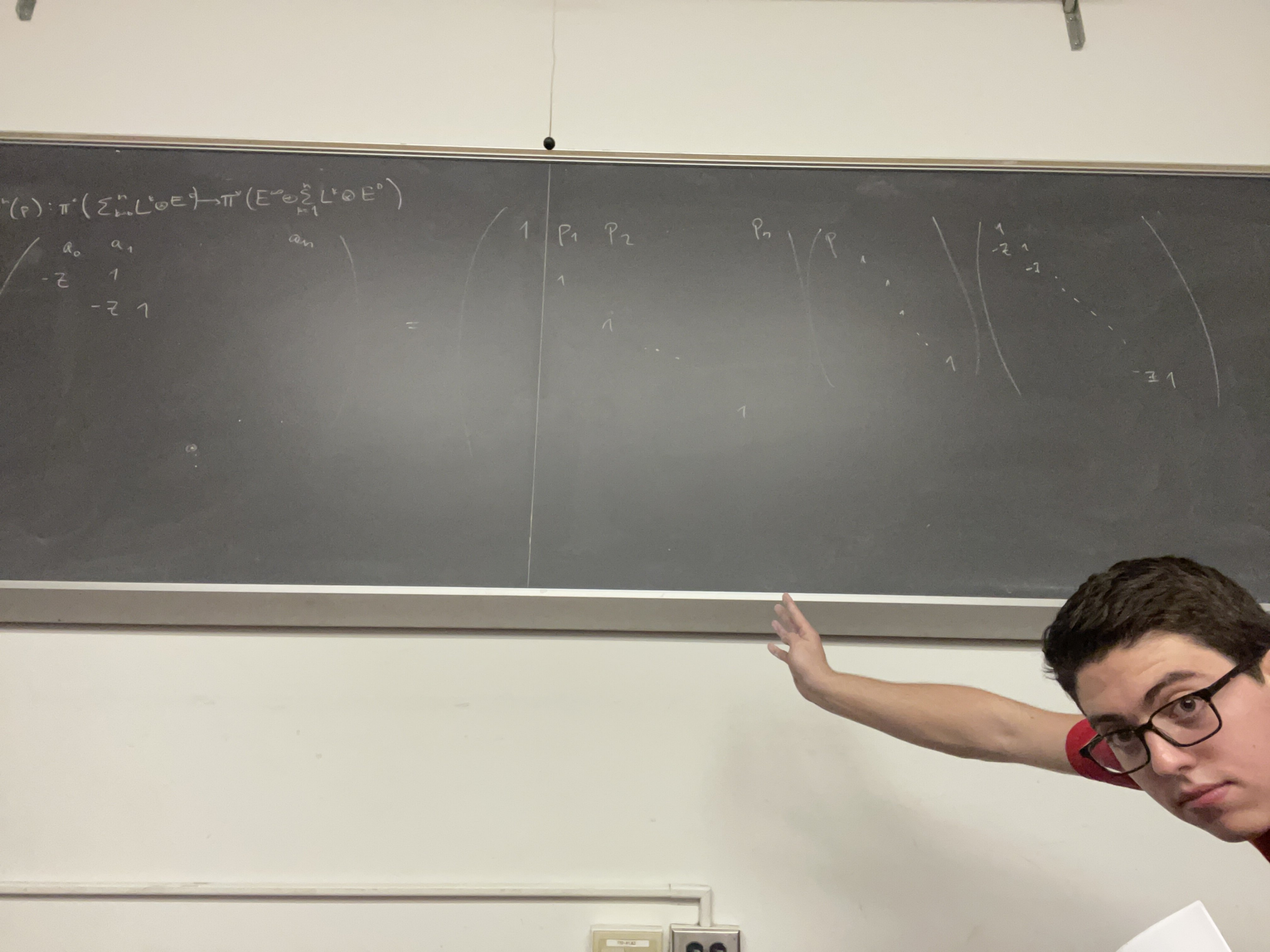
clothing

for $(E^{\bullet}$ [illegible]

$\rightsquigarrow$

$\mathcal{L}^n(P) : \pi^*\left(\Sigma_{k=0}^{n} L^k \otimes E^q\right) \longrightarrow \pi^*\left(E^{\infty} \oplus \sum_{k=1}^{n} L^k \otimes E^0\right)$

$$\begin{pmatrix} a_0 & a_1 & & & a_n \\ -z & 1 & & & \\ & -z & 1 & & \\ & & & & \end{pmatrix}$$


$^n(p) : \pi^*(\Sigma_{k=0}^n L^k \otimes E^0) \to \pi^*(E^\infty \oplus \sum_{k=1}^n L^k \otimes E^0)$
$a_0$ $a_1$ $a_n$
$-z$ $1$
$-z$ $1$
$=$
$1$ $p_1$ $p_2$ $p_n$
$1$
$1$
$1$
$p$
$1$
$1$
$1$
$1$
$-z$ $1$
$-z$ $1$
$-z$ $1$

$P_0 = P \quad \partial \; z P_{r-1}(z) = P_r(z) - P_r(0)$

$$\begin{pmatrix} 1 & & & \\ P_1 & 1 & & \\ P_2 & & \ddots & \\ P_n & & & 1 \end{pmatrix} \begin{pmatrix} P & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & \end{pmatrix} \begin{pmatrix} 1 & & & \\ -z & 1 & & \\ & -z & \ddots & \\ & & \ddots & 1 \\ & & & -z \end{pmatrix}$$

$$P_0 = P \quad \partial \; zP_{r-1}(z) = P_r(z) - P_r(0)$$

$$\begin{pmatrix} a_0 & a_1 & & \\ -z & 1 & & \\ & -z & 1 & \\ & & \ddots & \ddots \end{pmatrix} \qquad \begin{pmatrix} 1 & P_1 & P_2 & \cdots & P_n \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} P & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & & \\ -z & 1 & & & \\ & -1 & & & \\ & & \ddots & & \\ & & & -z & 1 \end{pmatrix}$$

$$\mathcal{L}^n(P) = (1+N_1)(P \otimes 1)(1+N_2)$$



$p_0 = p$ & $z p_{r+1}(z) = p_r(z) - p_r(0)$
$\mathcal{L}^n(p) = (1+N_1)(p \oplus 1)(1+N_2)$
$+ N_2$ are nilpotent.