

Quiz on manifolds

Name: _____

October 7, 2026

The following quiz is meant to test your understanding of the material so far and help me decide which topics need review. If you have no idea how to solve a problem, please indicate it clearly.

Instructions: Answer the following questions in class, focusing on the underlined parts. Record your answers on the back of this page and submit them to me before leaving class. You will then have until October 21st (two weeks from now) to solve them completely. You may use your personal notes, the class notes, and any of the listed references for help. Do not ask LLMs to solve the problems for you—it defeats the purpose of this quiz. Collaboration is allowed, but you must submit neatly handwritten final solutions in your own words. On October 21st, each student will be given one minute at the board to present their solution to one of the problems. After that lecture, full solutions will be posted.

1. Define smooth manifolds and smooth maps.
2. Show that S^2 is a smooth manifold by constructing explicit charts.
3. State the level set theorem and use it to construct TS^2 .
4. Give an example of a non-trivial smooth vector field on the torus. Compute its integral curves and its flow.
5. Define and use the standard basis to describe all differential forms on $\mathbf{R}^4$.
6. Let $\omega = z^2 dx \wedge dy + w dw \wedge dz$ be a 2-form on $\mathbf{R}^4$. Define $s : \mathbf{R}^3 \rightarrow \mathbf{R}^4$ by

$$\mathbf{x} = (x_1, x_2, x_3) \mapsto \left(\frac{2x_1}{|\mathbf{x}|^2 + 1}, \frac{2x_2}{|\mathbf{x}|^2 + 1}, \frac{2x_3}{|\mathbf{x}|^2 + 1}, \frac{|\mathbf{x}|^2 - 1}{|\mathbf{x}|^2 + 1} \right).$$

Compute $d\omega$ and $s^*\omega$.

7. State Stokes' theorem. Let η be the 3-form $z dx \wedge dy \wedge dz$ restricted from $\mathbf{R}^4$ to S^3 . Compute $\int_{S^3} \eta$. Does η vanish anywhere? If so, where?
8. Define the gradient of a function on a Riemannian manifold. Compute the gradient of $\sin : S^1(R) \rightarrow \mathbf{R}$ where $S^1(R)$ is the circle of radius R with the standard metric.