Homotopy limits + Colimits

M simplicially enriched

hM has $hM(x,y) = \pi_0 M(x,y)$

product $\prod m_\alpha$ given by m

$$M(-,m) \xrightarrow{\sim} \prod M(-,m_\alpha)$$

given by

$$\begin{tikzcd} x \arrow[d, "\exists !"'] \arrow[dr, Rightarrow] & \\ m \arrow[r] & m_\alpha \end{tikzcd}$$

def htpy prod ~~m~~, m st

$$Map(x,m) \longrightarrow \prod_\alpha Map(x, m_\alpha)$$

"natural weak htpy eq $\forall x$" — vs. M?

Ex show this is product in hM: apply π_0 to ~~Yoneda~~.

Thm (Vogt) M cocomplete enriched + tensored over spaces

then hM has all weak colims. — cotensor.

$F: D \to hM$ has a cone "non unique"

$$M(V \otimes M, N) \cong M(M, \{V, N\}) \cong M(V, Map(M,N))$$

what is this?

idea: colims ~~as~~ computed as: ?

$$\coprod_{a,b \in D} D(a,b) \times Fa \;\underset{proj}{\overset{ev}{\underset{id}{\rightrightarrows}}}\; \coprod_{a \in D} Fa$$

as sets?

pf lift $F: D \to hM$ to $F: D \to M$

$$\left(\coprod_{a,b} D(a,b) \times I \times Fa\right) \amalg \left(\coprod_{a\in} Fa\right)$$

source

mod

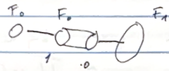
$$\uparrow \; \iota \amalg \iota \amalg id \qquad\qquad ev \amalg proj \amalg proj$$

$$\coprod \left(D(a,b) \times \{0\} \times Fa \amalg D(a,b) \times \{1\} \times Fa \right) \amalg \coprod I \times Fa$$

$D: \cdot \to \overset{1}{\cdot} \quad \longrightarrow \quad F_0 \xrightarrow{Ff} F_1$

$$\left(\{f\} \times I \times F_0\right) \amalg \left(F_0 \amalg F_1\right)$$

$$\underbrace{f \times 0 \times F_0}_{\substack{\cong \\ F_1}} \amalg \underbrace{f \times 1 \times F_0}_{\substack{\cong \\ F_0}} \amalg \underbrace{I \times F_0}_{\substack{\cong \\ F_0}} \amalg \underbrace{I \times F_1}_{\substack{\cong \\ F_1}} \quad ?$$



wcolim is a colim b/c cone is $D \times 2 \to hM$
lift to $D \times 2 \to M$
& use UP of pushout.

Def 5.2.2. homotopy colim D small cat
left derived $\mathbb{L}\mathrm{colim} : M^D \to M$
$\mathbb{R}\lim : M^D \to M$

$$\begin{tikzcd} M^D \arrow[r, "\gamma^D"] \arrow[d, "\gamma"] & (\mathrm{Ho}M)^D \\ \mathrm{Ho}(M^D) \arrow[ur, dashed, "\exists !"'] & \end{tikzcd}$$

when do these differ?